SIMONA TOPOR POP

MATH 121
HOMEWORK 11
SOLUTIONS

7.2.3 (ex. (b))

(a) $(t-2)^5(t-3)^2$

(c) Only for $\lambda_1 = 3$

(d) For $\lambda_1 = 3$, $p_1 = 1$
For $\lambda_2 = 2$, $p_2 = 3$

(e) (i) $\begin{cases} \text{rank } U_1 = 0 \\ \text{rank } U_2 = 3 \end{cases}$ (ii) $\begin{cases} \text{rank } U_1^2 = 0 \\ \text{rank } U_2^2 = 1 \end{cases}$ (iii) $\begin{cases} \text{null. } U_1 = 2 \\ \text{null } U_2 = 2 \end{cases}$ (iv) $\begin{cases} \text{null } U_1^2 = 2 \\ \text{null } U_2^2 = 4 \end{cases}$

by Thm. 7.10

by dimension formula, of course!

7.2.4 – See ★

(b) $A = \begin{pmatrix} 0 & 1 & -1 \\ -4 & 4 & -2 \\ -2 & 1 & 1 \end{pmatrix}$

$f(t) = -(t-1)(t-2)^2 \Rightarrow \lambda_1 = 1$ (mult. 1)
$\lambda_2 = 2$ (mult. 2)

$E_1 = N(A-I) = \text{Span}\left\{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}\right\}$; $E_2 = N(A-2I) = \text{Span}\left\{\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}\right\}$

So: $Q = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix}$ and $J = Q^{-1}AQ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & -1 & -1 \\ -3 & -1 & -2 \\ 7 & 5 & 6 \end{bmatrix}$ $f(t) = -(t-1)(t-2)^2 \rightarrow \begin{cases} \lambda_1 = 1 \text{ (mult. 1)} \\ \lambda_2 = 2 \text{ (mult. 2)} \end{cases}$ (again) !

$E_1 = N(A-I) = \text{Span}\left\{\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\right\}$; $N(A-2I) = \text{Span}\left\{\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}\right\}$

(Go on to $N(A-2I)^2 = \text{Span}\left\{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} +2 \\ +1 \\ 0 \end{pmatrix}\right\}$