MATH 121 : FIRST HOMEWORK

This homework is designed to refresh your memory of some prerequisite material. It is due in class on Tuesday 10th February, and will be graded on a completion-only basis (*i.e.* if you want, you can turn in a sheet of paper with only your name on it and you will receive full credit.)

(1) (Gaussian elimination) Find all solutions to the following equations:

(a)

$$\begin{aligned} 3x + 2y + 3z &= 0 \\ y + z &= 0 \end{aligned}$$

(b)

$$\begin{aligned} 3x + 2y + 3z &= 5 \\ y + z &= 2 \end{aligned}$$

(c)

$$\begin{aligned} 2x + y + z &= 4 \\ x + 2z &= 3 \end{aligned}$$

(2) (Bases) Which of the following are bases for $\mathbb{R}^3$?

(a)

$$\{\langle 1, 0, 0\rangle, \langle 0, 1, 0\rangle, \langle 0, 0, 1\rangle\}$$

(b)

$$\{\langle 1, 0, 0\rangle, \langle 1, 2, 1\rangle, \langle 3, 2, 1\rangle\}$$

(c)

$$\{\langle 1, 0, 0\rangle, \langle 1, 1, 0\rangle, \langle 1, 1, 1\rangle, \langle 4, 5, 6\rangle\}$$

(d)

$$\{\langle 0, 1, -1\rangle, \langle 0, 1, -2\rangle, \langle 1, -1, 1\rangle\}$$

(3) (Bases and dimension) Find a basis for the set of anti-symmetric (also known as *skew-symmetric*) 2×2 matrices whose upper-left entry is zero. What is the dimension of this space?

(4) (Dimension) Compute the dimensions of the following spaces:

(a)

$$\operatorname{span}\{\langle 1, 0, 1\rangle, \langle 0, 1, -2\rangle, \langle 2, 1, 0\rangle\}$$

(b) The space of symmetric 2×2 matrices.

(5) (Eigenvalues and eigenvectors) Find the eigenvalues and eigenvectors of the following matrices:

(a)

$$\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 2 & 0 & -1 \\ 4 & 1 & -4 \\ 2 & 0 & -1 \end{pmatrix}$$