$$= \det\left([T^t]_{\beta^*}^{\beta^*} - tI \right)$$

$$= \text{char poly of } T^t \qquad \square$$

Thus each eigenvalue of T is also an eigenvalue of T^t (and vice versa) and the corresponding algebraic multiplicities are equal.

Looking at our necessary and sufficient conditions for diagonalizability, we see that in order to show that T is diagonalizable $\iff$ T^t is diagonalizable, it suffices to show:

<u>Claim</u>: Let λ be an eigenvalue for T (and hence for T^t).
Let E_λ be the eigenspace for T with eigenvalue λ.
Let E'_λ be the eigenspace for T^t with eigenvalue λ.
Then
$$\dim E_\lambda = \dim E'_\lambda$$

<u>Proof</u>:
$$\begin{aligned}
\dim E'_\lambda &= \dim N(T^t - \lambda I) \\
&= \dim N(T-\lambda I)^t \\
&= \dim\left(R(T-\lambda I)^\circ \right) && \text{from part (a)} \\
&= \dim V - \dim R(T-\lambda I) && \text{from Q5 on midterm 1}
\end{aligned}$$