Claim: The solution space to $y'' - 4y = -2$ is

$$\{ y = \cos^2 t + ae^{2t} + be^{-2t} : a, b \in \mathbb{R} \}$$

Proof y is a solution to $y'' - 4y = -2$

$\Updownarrow$

$y - \cos^2 t$ is a solution to $y'' - 4y = 0$

$\Updownarrow$

$y - \cos^2 t = ae^{2t} + be^{-2t}$ for some $a, b \in \mathbb{R}$.

□

3. In this question, I will use the textbook's notation for quotient spaces.

Claim: The map $S : V \longrightarrow V/W$

$v \longmapsto v + W$

is (a) linear

(b) onto

Proof (a) $S(v+v') = v + v' + W$

$= (v+W) + (v'+W)$

$= S(v) + S(v')$ for all $v, v' \in V$

$S(cv) = cv + W$

$= c(v+W) = cS(v)$ for all $v \in V$, $c \in \mathbb{C}$

(b) Any element in V/W is of the form $v + W$ for some $v \in V$, so S is onto.

□