which is 2-dimensional. The eigenspace corresponding to the eigenvalue 2 is at least one-dimensional (since it is non-empty) and hence is exactly one-dimensional (since algebraic multiplicity is always $\geq$ geometric multiplicity).
Thus:

- ★ the characteristic polynomial of T splits
- ★ for each eigenvalue, the algebraic and ~~eg~~ geometric multiplicities are equal

Thus T is diagonalizable.

2. Let $\{y_1, \ldots, y_r\}$ be a basis for $N(T)$
$\{x_1, \ldots, x_s\}$ be a basis for $N(\mathcal{U})$

Since $\mathcal{U}$ is onto, we can find $v_1, \ldots, v_r \in V$ such that $\mathcal{U}(v_1) = y_1, \ldots, \mathcal{U}(v_r) = y_r$.

<u>Claim</u>: $\beta = \{v_1, \ldots, v_r, x_1, \ldots, x_s\}$ is a basis for $N(T\mathcal{U})$

<u>Proof</u>: Since $T\mathcal{U}(v_i) = T(y_i) = 0$
and $T\mathcal{U}(x_j) = T(0) = 0$
we know that $\beta \subseteq N(T\mathcal{U})$

Suppose that $a_1 v_1 + \cdots + a_r v_r + b_1 x_1 + \cdots + b_s x_s = 0$

Then, applying $\mathcal{U}$, we find that $a_1 y_1 + \cdots + a_r y_r = 0$ and so $a_1 = a_2 = \cdots = a_r = 0$ (because the set $\{y_1, \ldots, y_r\}$ is LI). Thus $b_1 x_1 + \cdots + b_s x_s = 0$, and hence $b_1 = b_2 = \cdots = 0$ (because the set $\{x_1, \ldots, x_s\}$ is LI). Therefore β is LI.