as otherwise we will have found eigenvectors v_1 and v_2 for distinct eigenvalues λ_1, λ_2 such that $v_1 + v_2 = 0$, contradicting part (a). Thus both v_1 and v_2 are zero (since if one is zero, so is the other), and so

$$a_1 x_1 + \cdots + a_k x_k = 0$$

$$b_1 y_1 + \cdots + b_{n-k} y_{n-k} = 0$$

Since β_1 and β_2 are LI we conclude that $a_1 = a_2 = \cdots = a_k = b_1 = \cdots = b_{n-k} = 0$, and so β is LI.
Thus β is a basis for V. $\square$

We have found a basis for V consisting of eigenvectors for T, so T is diagonalizable