$\{v_1, \ldots, v_k\}$ is LI.

By induction, we are done.

(b) Pick a basis $\beta_1 = \{x_1, \ldots, x_k\}$ for E_{λ_1}

and a basis $\beta_2 = \{y_1, \ldots, y_{n-k}\}$ for E_{λ_2}.

It suffices to prove:

<u>Claim</u>: $\beta = \beta_1 \cup \beta_2 = \{x_1, \ldots, x_k, y_1, \ldots, y_{n-k}\}$ is a basis for V consisting of eigenvectors for T.

<u>Proof</u>: Since $x_i \in E_{\lambda_1}$, $T(x_i) = \lambda_1 x_i$.

Since $y_j \in E_{\lambda_2}$, $T(y_j) = \lambda_2 y_j$.

Thus β consists of eigenvectors for T. Also, since $\lambda_1 \neq \lambda_2$ none of the x_i are equal to any of the y_j, so β consists of n vectors. To show β is a basis for V, which is n-dimensional, we just need to show β is LI.

Suppose that $a_1 x_1 + \cdots + a_k x_k + b_1 y_1 + \cdots + b_{n-k} y_{n-k} = 0$

Put $v_1 = a_1 x_1 + \cdots + a_k x_k$
$v_2 = b_1 y_1 + \cdots + b_{n-k} y_{n-k}$

Then $T(v_1) = \lambda_1 v_1$ and $T(v_2) = \lambda_2 v_2$ and $\lambda_1 \neq \lambda_2$, so at least one of v_1 and v_2 must be zero