$$= \dim N(T - \lambda I) \qquad \text{(Dimension Th}^{\underline{m}}\text{!)}$$

$$= \dim E_\lambda . \qquad \square$$

5. (a) We prove this by induction on k.

<u>Base case</u> : $k = 1$

We need to show that $\{v_1\}$ is LI. But v_1 is an eigenvector, so by definition is non-zero. Thus $\{v_1\}$ is LI

<u>Induction step</u> : Assume that the result is true for $k-1$, so that $\{v_1, \ldots, v_{k-1}\}$ is LI.

if $a_1 v_1 + \cdots + a_k v_k = 0 \quad \cdots \text{(1)}$

then $T(a_1 v_1 + \cdots + a_k v_k) = 0$

$$\Rightarrow \quad a_1 \lambda_1 v_1 + \cdots + a_k \lambda_k v_k = 0 \quad \cdots \text{(2)}$$

<u>(2) $- \lambda_k$ (1)</u> : $a_1(\lambda_1 - \lambda_k) v_1 + \cdots + a_{k-1}(\lambda_{k-1} - \lambda_k) v_{k-1} = 0$

and, since $\{v_1, \ldots, v_{k-1}\}$ is LI, we have that

$$a_1(\lambda_1 - \lambda_k) = 0, \ldots, a_{k-1}(\lambda_{k-1} - \lambda_k) = 0$$

Since all the λ_i are distinct, $a_1 = a_2 = \cdots = a_{k-1} = 0$ and plugging this in to (1) we find (since $v_k \neq 0$) that $\lambda_k = 0$ also. Thus