

So g_1, g_2 and h are linear.

(b) Suppose that $ag_1 + bg_2 = 0$ ----- (1)

Apply this equality to the function $f(x) = x$ to get

$$a g_1(x) + b g_2(x) = 0(x)$$

$$\Rightarrow \quad a + 2b = 0 \quad \text{----} (2)$$

Applying equality (1) to the function $f(x) = x^2$, we find

$$a g_1(x^2) + b g_2(x^2) = 0(x^2)$$

$$\Rightarrow \quad a + 4b = 0 \quad \text{---} (3)$$

From equations (2) and (3) we find that $a = b = 0$.

Thus $\{g_1, g_2\}$ is LI.

(c) Suppose that $ag_1 + bg_2 = h$ for $a, b \in \mathbb{R}$

Then

$$a g_1(x) + b g_2(x) = h(x) \rightsquigarrow a + b = \tfrac{1}{2} \quad \text{---} (4)$$

$$a g_1(x^2) + b g_2(x^2) = h(x^2) \rightsquigarrow a + 4b = 1/3 \quad \text{---} (5)$$

$$a g_1(x^3) + b g_2(x^3) = h(x^3) \rightsquigarrow a + 8b = 1/4 \quad \text{--} (6)$$

Equations (4), (5) and (6) are inconsistent, so h is not in the span of $\{g_1, g_2\}$.