Q4 (a) I need to show that g_1, g_2 and h are linear. Let f and $\tilde{f}$ be arbitrary elements of V and let $c \in \mathbb{R}$ be an arbitrary scalar.

Then:

$$(\ast) \quad g_1(f+\tilde{f}) = (f+\tilde{f})(1) = f(1) + \tilde{f}(1) = g_1(f) + g_1(\tilde{f})$$

$$(\ast) \quad g_1(cf) = (cf)(1) = c\,f(1) = c\,g_1(f)$$

$$(\ast) \quad g_2(f+\tilde{f}) = (f+\tilde{f})(2) = f(2) + \tilde{f}(2) = g_2(f) + g_2(\tilde{f})$$

$$(\ast) \quad g_2(cf) = (cf)(2) = c\,f(2) = c\,g_2(f)$$

$$(\ast) \quad h(f+\tilde{f}) = \int_0^1 (f+\tilde{f})(x)\,dx = \int_0^1 f(x)+\tilde{f}(x)\,dx = \int_0^1 f(x)\,dx + \int_0^1 \tilde{f}(x)\,dx = h(f) + h(\tilde{f})$$

$$(\ast) \quad h(cf) = \int_0^1 c f(x)\,dx = c\int_0^1 f(x)\,dx = c\,h(f)$$