W_1 is not a subspace as it is not closed under addition: if $f \in W_1$ and $g \in W_1$ then $f(1) = 1$ and $g(1) = 1$, so

$$(f+g)(1) = f(1) + g(1)$$
$$= 2$$

so $f+g \notin W_1$.

(c) Consider the map $ev_1 : P_n(\mathbb{R}) \to \mathbb{R}$

$$f \mapsto f(1)$$

This map is linear (we proved this in class) and $W_0 = N(ev_1)$.

Further, the range $R(ev_1)$ of ev_1 is equal to all of $\mathbb{R}$, because

$$ev_1(a) = a \qquad \text{for all } a \in \mathbb{R}$$

constant polynomial a

$\therefore$ rank$(T) = 1$

By the Dimension Theorem,

$$\dim(W_0) = \text{nullity}(T) = (n+1) - \text{rank}(T)$$
$$= n$$