so $$[T]_\beta^\gamma = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

(b) Let us first compute $N(T)$. If

$T(a+bx+cx^2) = 0$ then $ax + \frac{1}{2}bx^2 + \frac{1}{3}cx^3 = 0$

$\Rightarrow a = b = c = 0$

$\therefore N(T) = \{0\}$

By the Dimension Theorem, $\text{rank}(T) + \text{nullity}(T) = 3$

$\Rightarrow \underline{\underline{\text{rank}(T) = 3}}$

Q3 <u>Dimension Theorem</u>: Let V be a finite-dimensional vector space, let W be a vector space (both over the same field $\mathbb{F}$) and let $T: V \to W$ be a linear transformation. Then

$$\text{rank}(T) + \text{nullity}(T) = \dim V$$

<u>Proof</u>: We know that $N(T)$ is a subspace of V, so pick a ~~subspace~~ basis $\{v_1, \dots, v_k\}$ for $N(T)$.

Extend this to a basis $\{v_1, \dots, v_k, v_{k+1}, \dots, v_{k+l}\}$ for V.

We know that if β is any basis for V then