(b) Let $V = P(\mathbb{R})$

$$T: V \longrightarrow V, \quad f \longmapsto f'$$

$$S: V \longrightarrow V, \quad f \longmapsto \int_0^x f(x)\,dx$$

Then $$TS(f) = \frac{d}{dx}\left(\int_0^x f(x)\,dx\right)$$

$$= f$$

so T is right-invertible.

Claim: T is not left-invertible

Proof: Otherwise we could find $S: V \to V$ with $ST = I_V$. Let a be a constant polynomial. Then

$$S(T(a)) = I_V(a)$$

$$= a$$

and $T(a) = 0$, so $S(0) = a$.

But a was arbitrary ✗ S is supposed to be a well-defined function, so should have only one value for $S(0)$.

So T is not left invertible.