SOLUTIONS TO PRACTICE MIDTERM

Q1 (a) Let β be the standard basis for $\mathbb{R}^2$.

Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation with matrix

$[T]_\beta^\beta = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ (so $T(x,y) = (y,0)$)

Let $\mathcal{U} = T$. Then neither $\mathcal{U}$ nor T is zero (since their matrices are non-zero)

but

$$[\mathcal{U}T]_\beta^\beta = [\mathcal{U}]_\beta^\beta [T]_\beta^\beta = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

so $\mathcal{U}T$ is the zero map.

So the statement is false.

(b) This is false. For example, take

$\beta_1 = \{(1,0,0)\}$, $\beta_2 = \{(2,0,0)\}$ (both subsets of $\mathbb{R}^3$)

Then $\operatorname{span}\beta_1 \cap \operatorname{span}\beta_2 =$ the x-axis

and $\operatorname{span}(\beta_1 \cap \beta_2) = \operatorname{span}\emptyset = \{0\}$.

(c) This is false. For example, take

$V = \operatorname{span}\{(1,0,0), (0,1,0)\}$ $W = \operatorname{span}\{(0,0,1)\}$

$Z = \operatorname{span}\{(1,1,1)\}$.