So

$$y = e^{at}\left(A\cos(bt) - B\sin(bt) + A\cos bt - B\sin bt\right)$$

$$= 2Ae^{at}\cos(bt) - 2Be^{at}\sin(bt)$$

In other words $y \in \text{span}\{e^{at}\cos bt, e^{at}\sin bt\}$

Since both $e^{at}\cos bt$ and $e^{at}\sin bt$ are in the solution space [take $A = C = \frac{1}{2}$, $B = D = 0$ or $A = C = 0$, $B = -\frac{1}{2}$, $D = \frac{1}{2}$] we see that the solution space in $C^\infty(\mathbb{R})$ is span $\{e^{at}\cos(bt), e^{at}\sin(bt)\}$.

(b) We know $y = Ae^{-t}\cos 3t + De^{-t}\sin(3t)$, and that

$y(0) = 0 \Rightarrow A = 0$

and $y'(0) = v \Rightarrow 3B = v$

so $y(t) = \frac{v}{3}e^{-t}\sin(3t)$.

As $t \to \infty$, $y(t) \to 0$ i.e. the amplitude of the oscillation decreases to zero with time.