and so $$[T^*]_{\tilde{\gamma}}^{\tilde{\beta}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{2\sqrt{45}}{\sqrt{19-10\sqrt{2}}} & 0 \end{pmatrix}$$

Thus $$[T^*T]_{\tilde{\beta}}^{\tilde{\beta}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{2\sqrt{45}}{\sqrt{19-10\sqrt{2}}} & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & \frac{2\sqrt{45}}{\sqrt{19-10\sqrt{2}}} \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{180}{19-10\sqrt{2}} \end{pmatrix}$$

To construct the Singular Value Decomposition of T, we should pick an o.n. basis of eigenvectors for T^*T.

So $\tilde{\beta}$ will do! (the matrix of T^*T w.r.t. $\tilde{\beta}$ is already diagonal, so $\tilde{\beta}$ is a basis of eigenvectors, and $\tilde{\beta}$ is orthonormal by construction).

Actually, we should re-order so that the non-zero eigenvalue corresponds to the <u>first</u> eigenvector:

$$\beta = \{ y_3, y_1, y_2 \}$$

Now $T(y_3) = \dfrac{2\sqrt{45}}{\sqrt{38-20\sqrt{2}}} \cdot 1$

and we set $g_1 = \dfrac{T(y_3)}{\|T(y_3)\|} = 1/\sqrt{2}$.