and we need to show that $\lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$

[as then T will be nilpotent by (a)].

This is true, but it is <u>much</u> harder to prove than I had thought (and certainly far too hard to appear on the final).

Firstly, note that

$$\lambda_1 = \cdots = \lambda_n = 0 \iff (x-\lambda_1)\cdots(x-\lambda_n) = x^n$$

$$\iff x^n - (\lambda_1 + \cdots + \lambda_n)x^{n-1} + (\lambda_1\lambda_2 + \lambda_1\lambda_3 + \cdots)x^{n-2} + \cdots + (\lambda_1\lambda_2\cdots\lambda_n) = x^n$$

$$\iff \left\{\begin{array}{l} \lambda_1 + \cdots + \lambda_n = 0 \\ \sum_{i<j} \lambda_i\lambda_j = 0 \\ \sum_{i<j<k} \lambda_i\lambda_j\lambda_k = 0 \\ \vdots \\ \lambda_1\lambda_2\cdots\lambda_n = 0 \end{array}\right.$$

these are called THE ELEMENTARY SYMMETRIC FUNCTIONS of $\lambda_1, \ldots, \lambda_n$.

So it suffices to show that

$$\lambda_1 + \cdots + \lambda_n = 0$$
$$\sum_{i<j} \lambda_i\lambda_j = 0$$
$$\vdots$$
$$\lambda_1\lambda_2\cdots\lambda_n = 0$$