degree 1. So $D = C\prod_{i>j}(x_i - x_j)$

where C has degree zero, i.e. is a constant.

We need to show this constant is equal to 1.

<u>Base case</u>: $n=2$

$$D(x_1, x_2) = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \end{vmatrix} = x_2 - x_1$$

$$= 1.(x_2 - x_1) \quad \text{so } C = 1 \checkmark$$

<u>Induction step</u>: We know that $D(x_1, \dots, x_n) = C\prod_{i>j}(x_i - x_j)$

and we are assuming that $D(x_1, \dots, x_{n-1}) = \prod_{\substack{i>j \\ i \leq n-1}}(x_i - x_j)$

We want to show that $C = 1$.

Put $x_1 = 0$. Then $D(0, x_2, \dots, x_n) = \begin{vmatrix} 1 & 0 & \cdots & 0 \\ 1 & x_2 & \cdots & x_2^{n-1} \\ \vdots & & & \\ 1 & x_n & \cdots & x_n^{n-1} \end{vmatrix}$

$$= \begin{vmatrix} x_2 & \cdots & x_2^{n-1} \\ \vdots & & \vdots \\ x_n & \cdots & x_n^{n-1} \end{vmatrix}$$

$$= x_2 x_3 \cdots x_n \begin{vmatrix} 1 & x_2 & \cdots & x_2^{n-1} \\ 1 & x_3 & \cdots & x_3^{n-1} \\ \vdots & & & \\ 1 & x_n & \cdots & x_n^{n-1} \end{vmatrix} = x_2 \cdots x_n \prod_{\substack{i>j \\ i \leq n \\ j \geq 2}}(x_i - x_j)$$

by the induction hypothesis

On the other hand, putting $x_1 = 0$ turns

$$C\prod_{i>j}(x_i - x_j) \quad \text{into} \quad C(x_2 - 0)\cdots(x_n - 0)\prod_{\substack{i>j \\ j \neq 1}}(x_i - x_j)$$

$$= C x_2 \cdots x_n \prod_{\substack{i>j \\ j \geq 2}}(x_i - x_j)$$

So $C = 1$. By induction on n we are done.