$\tilde{T}^* = (T|_{W^\perp})^* = T^*|_{W^\perp}$ so $\tilde{T}$ is

normal :
$$\begin{aligned}\tilde{T}\tilde{T}^* &= T|_{W^\perp} T^*|_{W^\perp} \\ &= (TT^*)|_{W^\perp} \\ &= (T^*T)|_{W^\perp} \\ &= T^*|_{W^\perp} T|_{W^\perp} \\ &= \tilde{T}^*\tilde{T}\end{aligned}$$

By the induction hypothesis, $\exists$ an o.n. basis for $\tilde{V}$ consisting of eigenvectors for $\tilde{T}$, call it $\{v_1, \ldots, v_{n-1}\}$. Then $\{v_1, \ldots, v_{n-1}, v\}$ is an o.n. basis for V consisting of e'vectors for T.

By induction, we are done $\square$

(e) $V = \mathbb{C}^2$, $T(x,y) = (ix, 3iy)$.

This has ~~eigenvectors~~ non-real eigenvalues, so T cannot be self adjoint, but $T^*(x,y) = (-ix, -3iy)$

so $T^*T(x,y) = \cancel{(x,y)}\ (x, 9y) = TT^*(x,y)$