

#6(d) We will prove this by induction on $n = \dim V$.

Base case : $n = 1$

This is trivial.

Induction step : Assume that for every $(n-1)$ -dim^y i.p. space \tilde{V} and every normal linear transformation $\tilde{T} : \tilde{V} \rightarrow \tilde{V}$, there is an o.n. basis for \tilde{V} consisting of eigenvectors for \tilde{T} .

Since we are working over \mathbb{C} , the char. poly. of T splits and so \exists an eigenvector $v \in V$ for T . Let $W = \text{span}\{v\}$.

W is T -invariant $\Rightarrow W^\perp$ is T^* -invariant (from class)
 W is also T^* -invariant, since v is an eigenvector for T^* (by (c)), so W^\perp is T^{**} -invariant $\Rightarrow W^\perp$ is T -invariant.

Let $\tilde{V} = W^\perp$ and $\tilde{T} = T|_{W^\perp}$.

\tilde{V} is an $(n-1)$ -dim^y inner product space, and