therefore differ only by a reordering of their Jordan blocks.

(b) Let $\gamma = \{1, x, x^2, x^3\}$ be the standard basis for $P_3(\mathbb{R})$. Then

$$[T]_\gamma^\gamma = \begin{pmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 6 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

The only eigenvalue of T is 2 and

$$N(T-2I) = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$N\left((T-2I)^2\right) = N\left(\begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^2 \right) = N\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = P_3(\mathbb{R})$$

Pick $v \in N((T-2I)^2)$, $v \notin N(T-2I)$: say $v = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

Then $\{(T-2I)v, v\} = \left\{ \begin{pmatrix} 0 \\ 6 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ is a cycle of gen. eigenvectors.

Now try to pick $w \in N(T-2I)^2$, ~~$w \notin \operatorname{span}\{(T-2I)v, v\}$~~

$w \notin \operatorname{span}\left(\{(T-2I)v, v\} \cup N(T-2I)\right) = \operatorname{span}\left\{ \begin{pmatrix} 0 \\ 6 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$.

Take, for example, $w = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$. Then $\{(T-2I)w, w\} = \left\{ \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$