$$= n_1(\lambda) + 2n_2(\lambda) + 2n_3(\lambda) + \cdots + 2n_k(\lambda) + \cdots$$

Repeating for $(T-\lambda I)^3, (T-\lambda I)^4, \ldots$ etc. proves the Claim. $\square$

Thus
$$n_1(\lambda) = 2\,\text{nullity}(T-\lambda I) - \text{nullity}(T-\lambda I)^2$$

$$n_2(\lambda) = \frac{3}{2}\left(\text{nullity}(T-\lambda I)^2 - n_1(\lambda)\right) - \left(\text{nullity}(T-\lambda I)^3 - n_1(\lambda)\right)$$

$$\vdots$$

and so the numbers $n_1(\lambda), n_2(\lambda), \ldots$ can be uniquely determined from the numbers

$$\text{nullity}(T-\lambda I),\ \text{nullity}(T-\lambda I)^2, \ldots$$

But these do not depend on the choice of Jordan canonical basis δ, in other words

$$[T]_\beta^\beta \quad \text{and} \quad [T]_\gamma^\gamma$$

have the same # of Jordan blocks of each size for each generalized eigenvalue λ. They