so nullity $(T-\lambda I)$ = # of blocks with generalized eigenvalue λ

$$= \underbrace{n_1(\lambda) + n_2(\lambda) + \cdots + n_k(\lambda) + \cdots}$$

this looks like an infinite sum but in fact isn't, as there are no blocks of size bigger than $\mathrm{mult}(\lambda)$, so $n_{k'}(\lambda) = 0$ for all $k' > \mathrm{mult}(\lambda)$.

Similarly, nullity $(T-\lambda I)^2$ = nullity $(A_1 - \lambda I)^2 + \cdots + $ nullity $(A_r - \lambda I)^2$

and nullity $(A_j - \lambda I)^2 = 0$ if $\lambda_j \neq \lambda$ [the diagonal entries of $(A_j - \lambda I)^2$ will be non-zero]

and it is upper-triangular

and nullity $(A_j - \lambda I)^2$ = nullity $\begin{pmatrix} 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & 0 & \vdots \\ & & & & \ddots & 0 \\ & & & & & 1 \\ & & & & & \end{pmatrix}$

$= 2$ if the block has size at least 2

and nullity $(A_j - \lambda I)^2$ = $\cancel{\text{nullity} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}^2}$

nullity $(0)^2 = 1$ if the block has size 1

So nullity $(T-\lambda I)^2 = 1 \cdot$ (# of blocks of size 1 for λ)

$+ 2 \cdot$ (# of blocks of size ≥ 2 for λ)