#5 : **Claim** : Let δ be a basis for V such that (10)

$[T]_\delta^\delta$ is in Jordan canonical form

with
$n_1(\lambda)$ blocks of size 1 with gen. e'value λ
$n_2(\lambda)$ blocks of size 2 with gen. e'value λ
$\vdots$
$n_k(\lambda)$ blocks of size k with gen. e'value λ
$\vdots$

Then
$$\text{nullity}\,(T-\lambda I) = n_1 + \cdots + n_k + \cdots$$
$$\text{nullity}\,(T-\lambda I)^2 = n_1 + 2n_2 + 2n_3 + \cdots + 2n_k + \cdots$$
$$\text{nullity}\,(T-\lambda I)^3 = n_1 + 2n_2 + 3n_3 + \cdots + 3n_k + \cdots$$
$$\vdots$$

Proof :
$$\text{nullity}\,(T-\lambda I) = \text{nullity}\,\left([T]_\delta^\delta - \lambda I\right)$$
$$= \text{nullity} \begin{pmatrix} A_1-\lambda I & O & O \\ O & \ddots & O \\ O & O & A_r-\lambda I \end{pmatrix}$$

where $A_1, \ldots, A_r$ are the Jordan blocks

$$= \text{nullity}\,(A_1-\lambda I) + \cdots + \text{nullity}\,(A_r-\lambda I)$$

But
$$\text{nullity}\,(A_j-\lambda I) = \text{nullity} \begin{pmatrix} \lambda_j-\lambda & 1 & & O \\ & \lambda_j-\lambda & 1 & \\ & & \ddots & 1 \\ O & & & \lambda_j-\lambda \end{pmatrix} = \begin{cases} 1 & \text{if } \lambda_j = \\ 0 & \text{else} \end{cases}$$