Q1 (a) $\operatorname{tr}(A) = \sum_{i=1}^{i=n} A_{ii}$

so $\operatorname{tr}(AB) = \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} A_{ij} B_{ji}$

$= \sum_{j=1}^{j=n} \sum_{i=1}^{i=n} B_{ji} A_{ij}$

$= \operatorname{tr}(BA)$

Matrices C and D are similar $\iff$ $C = Q^{-1} D Q$ for some invertible matrix Q

so if C and D are similar then $\operatorname{tr}(C) = \operatorname{tr}(Q^{-1}DQ)$
$= \operatorname{tr}(DQQ^{-1})$
$= \operatorname{tr}(D)$

(b) Put
$$D(x_1, \ldots, x_n) = \begin{vmatrix} 1 & x_1 & \cdots & x_1^{n-1} \\ 1 & x_2 & \cdots & x_2^{n-1} \\ \vdots & & & \\ 1 & x_n & \cdots & x_n^{n-1} \end{vmatrix}$$

If $x_i = x_j$ for any $i \neq j$ then the determinant will vanish (as two of the rows will become equal)

so D is divisible by $\prod_{i>j} (x_i - x_j)$

If we let each x_i have degree 1 then $D(x_1, \ldots, x_n)$ has degree $0 + 1 + \ldots + n-1 = \frac{1}{2}n(n-1)$

every entry in the first column has degree zero

every entry in the second col. has degree 1

every entry in the last column has degree $n-1$

and in expanding the determinant we take exactly one entry from each column

But the degree of $\prod_{i>j} (x_i - x_j)$ is also $\frac{1}{2}n(n-1)$
[it is the product of $\frac{1}{2}n(n-1)$ terms, each of which has