

# CYCLIC SUBSPACES

⑧

Let  $T: V \rightarrow V$  be a linear map and

let  $v \in V$ . The subspace

$$W = \text{span} \{ v, T(v), T^2(v), \dots \}$$

is called the  $T$ -cyclic subspace of  $V$

generated by  $v$

Claim:  $W$  is  $T$ -invariant

Proof: If  $w \in W$  then

$$w = a_1 T^{i_1}(v) + \dots + a_n T^{i_n}(v)$$

for some  $i_1, \dots, i_n$  and scalars  $a_1, \dots, a_n$ .

Then

$$T(w) = a_1 T^{i_1+1}(v) + \dots + a_n T^{i_n+1}(v)$$

$\in W$

(by linearity of  $T$ )

□