clear rows using column operations

$$\det\left(\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline 0 & \tilde{C} \end{array}\right) = \det\left(\begin{array}{c|c} \begin{matrix} \lambda_1 & * \cdots & * \\ & \ddots & * \\ 0 & & \lambda_k \end{matrix} & \tilde{B} \\ \hline 0 & \tilde{C} \end{array}\right)$$

$$= \det\left(\begin{array}{c|c} \begin{matrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_k \end{matrix} & 0 \\ \hline 0 & \tilde{C} \end{array}\right)$$

$$= \lambda_1 \lambda_2 \cdots \lambda_k \det(C)$$

$$= (\det \tilde{A})(\det \tilde{C}). \qquad \square$$

The textbook uses this result to prove the Cayley-Hamilton theorem:

<u>Th$^{\underline{m}}$</u> (5.23) Let $T: V \to V$ be a linear map, where V is a finite-dimensional vector space.

Let $f(t) = a_0 + a_1 t + \cdots + (-1)^n t^n$ be the characteristic polynomial of T. Then $f(T) = 0$, i.e.

$$a_0 I + a_1 T + \cdots + a_{n-1} T^{n-1} + (-)^n T^n$$

is the zero transformation.

Read their proof of this now; we will prove it a little later in the course, once we know about Jordan Canonical Form.