this by, for example, observing that

(a) if $\det \tilde{A} = 0$ then $N(\tilde{A}) \neq \{0\}$ so
$\exists\ v \in N(\tilde{A}),\ v \neq 0$.

But then
$$\left(\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline 0 & \tilde{C} \end{array}\right)\begin{pmatrix} v \\ \hline \vec{0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad N\left(\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline 0 & \tilde{C} \end{array}\right) \neq \{0\}$$

$$\Rightarrow \quad \det\left(\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline 0 & \tilde{C} \end{array}\right) = 0$$

and (b) if $\det \tilde{A} \neq 0$ then we can ~~pick our basis [illegible] such that A~~ change basis in W such that $\tilde{A}$ becomes upper-triangular (by e.g. midterm 2 Q4), and since none of the entries on the diagonal of $\tilde{A}$ are zero we can use column operations (adding scalar multiples of the first k columns to other columns) to get rid of $\tilde{B}$ and the ~~uppe~~ non-diagonal parts of $\tilde{A}$: