T to W. Then $g(t)$ divides $f(t)$.

<u>Proof</u>: Pick a basis $\gamma = \{w_1, \dots, w_k\}$ for W and extend it to a basis $\beta = \{w_1, \dots, w_k, v_1, \dots, v_l\}$ for V. Then

$$[T]_\beta^\beta = \begin{array}{c} \\ k \\ l \end{array}\begin{array}{c} \begin{array}{cc} k & l \end{array} \\ \left(\begin{array}{c|c} A & B \\ \hline O & C \end{array}\right) \end{array}$$

where $A = [T_W]_\gamma^\gamma$.

Thus $f(t) = \det \left(\begin{array}{c|c} A - tI & B \\ \hline O & C - tI \end{array}\right)$

$$= \det(A - tI)\,\det(C - tI)$$

$$= g(t) \cdot \det(C - tI)$$

and so $g(t)$ divides $f(t)$. $\square$

this step follows from the fact that $\det \left(\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline O & \tilde{C} \end{array}\right) = (\det \tilde{A})(\det \tilde{C})$. One can see