Since W is T-invariant, the restriction T_W of T to W makes sense:

$$T_W : W \longrightarrow W$$
$$w \longmapsto T(w)$$

Let us compute the matrix $[T_W]_\beta^\beta$ of T_W with respect to the basis

$$\beta = \{v, T(v), \ldots, T^{k-1}(v)\}$$

Since β is a basis for W and $T^k(v) \in W$ we can find scalars $a_0, a_1, \ldots, a_{k-1}$ with

$$a_0 v + a_1 T(v) + \cdots + a_{k-1} T^{k-1}(v) + T^k(v) = 0$$

[warning: these aren't the same a_i as on pp 11–12]