<u>Induction step</u>: assume $T^j(v), T^{j+1}(v), \dots, T^{N-1}(v)$ are all in span γ. We need to show $T^N(v) \in \text{span}(\gamma)$

But

$$T^{N-1}(v) = b_0 v + b_1 T(v) + \dots + b_{j-1} T^{j-1}(v)$$

by the induction hypothesis, so

$$T^N(v) = b_0 T(v) + b_1 T^2(v) + \dots + b_{j-1} T^j(v)$$

$$= -\frac{b_{j-1} a_0}{a_j} v + \left(b_0 - \frac{b_{j-1} a_1}{a_j}\right) T(v) + \dots$$

$$+ \left(b_{j-2} - \frac{b_{j-1} a_{j-1}}{a_j}\right) T^{j-1}(v)$$

here we used the fact that $T^j(v) = -{}^{a_0}/_{a_j} v - \dots - \frac{a_{j-1}}{a_j} T^{j-1}(v)$

and so $T^N(v) \in \text{span}(\gamma)$

By induction, we are done. □

This proves the Subclaim, which shows that γ spans W. So γ is a basis for W. But $\dim W = k$ and γ has ~~k~~ j elements, so $j = k$ (which is what we wanted). □