<u>Subclaim</u>: $T^N(v) \in \operatorname{span} \gamma$ for all N

<u>Proof of Subclaim</u>:

This is obvious for $N = 0, 1, 2, \ldots, j-1$.
We prove it for $N \geq j$ by induction on N

<u>Base case</u> $N = j$

The set $\{v, T(v), \ldots, T^{j-1}(v), T^j(v)\}$ is linearly dependent (look at the definition of j!) so we can find scalars $a_0, \ldots, a_j$ with

$$a_0 v + a_1 T(v) + \cdots + a_j T^j(v) = 0$$

and not all the $a_i = 0$.
But a_j cannot be zero, as otherwise

$$a_0 v + a_1 T(v) + \cdots + a_{j-1} T^{j-1}(v) = 0$$

$\Rightarrow$ $a_0 = a_1 = \cdots = a_{j-1} = 0$ too (as γ is LI)

$\times\!\!\times$

So $a_j \neq 0$, and

$$T^j(v) = -\frac{a_0}{a_j} v - \frac{a_1}{a_j} T(v) - \cdots - \frac{a_{j-1}}{a_j} T^{j-1}(v)$$