If $j = 0$ then $v = 0$ and $W = \{0\}$, so the Theorem is true.

Otherwise $j \geq 1$; we know that $j \leq k$ as $\dim W = k$, and we want to show that $j = k$.

Claim: $\gamma = \{v, T(v), \ldots, T^{j-1}(v)\}$ is a basis for W

Proof: It is LI by definition. We need to show it spans W.

Since $W = \text{span}\{v, T(v), T^2(v), \ldots\}$ it suffices to show that $T^N(v) \in \text{span}(\gamma)$ for all N. This is because any $w \in W$ can be written as a LC of $v, T(v), \ldots$ so if each $T^N(v)$ can be written as a LC of elements of γ then each $w \in W$ can also be written as a LC of elements of γ.