

Now:

$$\begin{aligned} \text{"} \implies \text{"} \quad T^t \text{ onto} &\Rightarrow R(T^t) = V^* \Rightarrow (R(T^t))^0 = \{0\} \text{ by (15)} \end{aligned}$$

$$\Rightarrow (R(T^t))^0 = N[(T^t)^t] = \{0\} \Rightarrow N(T) = \{0\} \Rightarrow \underline{T \text{ is one-to-one}} \quad \checkmark$$

$$\text{"} \longleftarrow \text{"} \quad \underline{T \text{ one-to-one}} \Rightarrow N(T) = \{0\} \Rightarrow N[(T^t)^t] = \{0\}$$

$$\Rightarrow (R(T^t))^0 = \{0\} \text{ by (15)}$$

Proceed the same way as in 13a to get  $R(T^t) = V^* \Rightarrow \underline{T^t \text{ is onto}}$   
G.E.D.  $\checkmark$

~~$$\begin{aligned} R(T^t) &= V^* \\ R[(T^t)^t] &= \{0\} \end{aligned} \Rightarrow R$$~~

This completes the proof.

Note. I omitted 1.3.31 because it is not on the list of topics for the midterm. I'll come back with solutions to it.