People did very well on this generally - you just set up the system as in Example 5 to obtain:

$T^t g_1 = -f_1 - 2f_2$

$T^t g_2 = f_1 + f_2$

Read off the matrix : $\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = [T^t]_{\gamma^*}^{\beta^*}$.

ⓒ $T(1) = (-1, 1)$, $T(x) = (-2, 1)$ $\Rightarrow [T]_\beta^\gamma = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

Then $\left[[T]_\beta^\gamma\right]^t = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$ by definition.

So this verifies the statement of theorem 2.25 !

$\boxed{2.6.13}$. $S^\circ = \{f \in V^* \mid f(x) = 0, \forall x \in S\}$ anihilator of set S

ⓐ S° subspace of V^*. ✓

① non-empty : Clearly, $0_V \in V^*$. ✓

ⓘⓘ closure under addition Let $f, g \in S^\circ$.

Then $(f+g)(x) = f(x) + g(x) = 0 + 0 = 0 \Rightarrow f+g \in S^\circ$. ✓

ⓘⓘⓘ closure under multiplication : $f \in S^\circ$, $c \in \mathbb{F}$.

Then $(cf)(x) = c \cdot [f(x)] = c \cdot 0 = 0 \Rightarrow cf \in S^\circ$ ✓

So $S^\circ \subset V^*$ is a subspace.

ⓑ $W \subset V$ subspace, $x \notin W \Rightarrow \exists f \in W^\circ$ st. $f(x) \neq 0$.

Let $\gamma = \{v_1, \ldots v_m\}$ be a basis for W.

Expand this to a basis $\beta = \{v_1, \ldots v_m, v_{m+1}, \ldots v_n\}$ for V. Let $x \notin W$.

$\Rightarrow \beta^* = \{f_1, \ldots f_k, f_{k+1} \ldots f_n\}$, $f_i(x_j) = \delta_{ij}$.

So: $f_{k+1}(x) = 1$, $\forall i = \overline{1, k}$.

$f_{k+1}(v_i) = 0$

Can write any $v \in V$ as $v = a_1 v_1 + \ldots + a_k v_k$.

$\Rightarrow f_{k+1}(v) = f(a_1 v_1 + \ldots + a_k v_k) = a_1 f(v_1) + \ldots + a_k f(v_k) = 0$

$\Rightarrow f_{k+1} \in W^\circ$. But $f_{k+1}(x) = 1 \neq 0$!