SIMONA TOPOR POP

HOMEWORK #7
SOLUTIONS

$\boxed{2.\cancel{6}\,3\ a}$

$V=\mathbb{R}^3$, $\beta=\{(1,0,1),(1,2,1),(0,0,1)\}$.

Solve the following system for e_1:

$$\begin{cases} 1 = f_1(1,0,1) = f_1(e_1+e_3) = f_1(e_1)+f_1(e_3) \\ 0 = f_1(1,2,1) = f_1(e_1+2e_2+e_3) = f_1(e_1)+f_1(2e_2)+f_1(e_3) \\ 0 = f_1(0,0,1) = f_1(e_3) \end{cases}$$

Obtain $f_1(e_1)=1$, $f_1(e_2)=-\frac{1}{2}$, $f_1(e_3)=0$. Hence $\boxed{f_1(x,y,z) = x-\frac{1}{2}y}$.

Solve similar systems for e_2 and e_3 to get

$\boxed{f_2(x,y,z) = \frac{1}{2}y}$, $\boxed{f_3(x,y,z) = -x+z}$.

$\boxed{2.\cancel{6}\,7}$ $V=P_1(\mathbb{R})$, basis $\beta=\{1,x\}$
$W=\mathbb{R}^2$, basis $\gamma=\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\}$.

(a) $f(a,b)=a-2b$ $(f\in W^*)$.

$$T^t(f) = f\,T(p(x)) = f[p(0)-2p(1),\ p(0)+p'(0)] \overset{\text{by defn.}}{=} p(0)-2(p(1)-2p(0)-2p'(0) = -p(0)-2p(1)-2p'(0)$$

$T^t(f)(a+b) = (-a-2b-2a-2b) = -3a-4b.$

(b) $[T^t]_{\gamma^*}^{\beta^*} = \left[[T^t g_1]_{\gamma^*}\ [T^t g_2]_{\gamma^*}\right]$.

$g_1(x,y)=x$, $g_2(x,y)=y$.