$\boxed{1.4.15}$ $S_1, S_2 \subset V$ subsets.

— $\text{span}(S_1 \cap S_2) \subset \text{span}(S_1) \cap \text{span}(S_2)$.

Let $u \in S_1 \cap S_2 \Rightarrow u = a_1 u_1 + a_2 u_2 + \_\_ + a_n u_n$ ① with $\begin{cases} a_i \in F \\ u_i \in S_1 \cap S_2, \\ \forall i = \overline{1,n}. \end{cases}$

② Since $u_i \in S_1 \cap S_2$, $\forall i = \overline{1,n}$, $u_i \in S_1$. Hence ① means that u is in the span of S_1 (since it's a linear combination of vectors in S_1)

③ Similarly, since $u_i \in S_1 \cap S_2 \Rightarrow u_i \in S_2$. Hence ① means that u is in the span of S_2. (" " " " " in S_2).

②, ③ $\Rightarrow$ $u \in \text{Span}(S_1)$ and $u \in \text{Span}(S_2) \Rightarrow u \in \text{Span}(S_1) \cap \text{Span}(S_2)$.

$\Rightarrow \text{span}(S_1 \cap S_2) \subset \text{Span}(S_1) \cap \text{Span}(S_2)$.

— example: $\text{Span}(S_1 \cap S_2) = \text{Span}(S_1) \cap \text{span}(S_2)$.

Let $S_1 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ $\Rightarrow$ $\text{span}(S_1 \cap S_2) =$ x-axis in $\mathbb{R}^3$.

$S_2 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$

$\text{Span}(S_1) = \mathbb{R}^3$
$\text{Span}(S_2) =$ x-axis $\Rightarrow$ $\text{Span}(S_1) \cap \text{Span}(S_2) =$ x-axis in $\mathbb{R}^3$.

— example: $\text{Span}(S_1 \cap S_2) \neq \text{Span}(S_1) \cap \text{Span}(S_2)$.

Let $S_1 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$, $S_2 = \left\{ \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \right\}$ $\Rightarrow S_1 \cap S_2 = \emptyset$.

$\text{Span}(S_1 \cap S_2) = \emptyset$.
$\text{Span}(S_1) \cap \text{Span}(S_2) =$ x-axis in $\mathbb{R}^3$.