V is not a vector space because (VS 4) fails.

From (VS 3), zero vector in V is $(0,1)$.

Take element $(a_1, 0)$. Need additive inverse for it. But:

$(a_1, 0) + (b_1, b_2) \overset{\text{defn.}}{=} (a_1 + b_1, 0) \neq (0,1)$.

Note that (VS 8) also fails. (Why?)

$\boxed{1.2.16}$ One way to do this is to cite Example 2 in Friedberg:

$M_{m\times n}(F)$ is a vector space, provided F is a field.

$F = \mathbb{Q}$ is indeed a field (you can check the field axioms easily or cite class example).

The other way is to show that scalar multiplication is well-defined and that (VS 6)—(VS 8) hold.

$\boxed{1.2.18}$ $V = \{(a_1, a_2) \mid a_1, a_2 \in \mathbb{C}\}$.

$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2)$.

$c(a_1, a_2) = (ca_1, a_2)$

V is not a vector space because (VS 1) fails.

$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2)$

$(b_1, b_2) + (a_1, a_2) = (b_1 + 2a_1, b_2 + 3a_2)$

not equal in general!

$\boxed{1.3.10}$ $\longrightarrow W_1 = \{(a_1, \ldots, a_n) \in F^n \mid a_1 + a_2 + \ldots + a_n = 0\}$ is a subspace.

Check:

- $0 \in W_1$. Take $0 = (0, 0, \ldots, 0)$ clearly $\in W_1$.
- closure under addition:

  $(a_1, \ldots, a_n) + (b_1, \ldots, b_n) = (a_1 + b_1, \ldots, a_n + b_n)$

  $(a_1 + b_1) + \ldots + (a_n + b_n) = (a_1 + \ldots + a_n) + (b_1 + \ldots + b_n) = 0 + 0 = 0$
- closure under multiplication:

  $c(a_1, \ldots, a_n) = (ca_1, \ldots, ca_n)$ $\quad ca_1 + \ldots + ca_n = c(a_1 + \ldots + a_n) = c \cdot 0 = 0$.

$\longrightarrow W_2 = \{(a_1, \ldots, a_n) \in F^n \mid a_1 + \ldots + a_n = 1\}$ is not a subspace

For instance, closure under multiplication fails:

$(a_1 + b_1) + \ldots + (a_n + b_n) = (a_1 + \ldots + a_n) + (b_1 + \ldots + b_n) = 1 + 1 = 2 \neq 1$!