SIMONA TOPOR POP

MATH 121

HOMEWORK #2 - SOLUTIONS

$\boxed{1.2.12}$ $V = \{f : \mathbb{R} \to \mathbb{R} \mid f(t) = f(-t)\}$ is a vector space.

- check addition & multiplication well-defined:
  - $\exists$ unique $f+g \in V$: $(f+g)(t) = f(t) + g(t) = f(-t) + g(-t) = (f+g)(-t)$
  - $\exists$ unique $af \in V$: $[af](t) = a[f(t)] = a[f(-t)] = af(-t)$.
- check the 8 axioms:

(VS1) $(f+g)(t) = f(t) + g(t) = g(t) + f(t) = (g+f)(t)$.

(VS2) $[(f+g)+h](t) = (f+g)(t) + h(t) = f(t) + g(t) + h(t) =$
$= f(t) + (g+h)(t) = [f+(g+h)](t)$.

(VS3) $f \equiv 0$ $(f(t) = 0, \forall t \in \mathbb{R})$. clearly an even function.
Then $g(t) + 0 = g(t)$

(VS4) Given $f(t)$, consider $g(t) = (-1)\cdot f(t) = -f(-t) = g(-t)$,
well-defined & unique, and $f(t) + g(t) = f(t) + (-1)f(t) =$
(even) $= (1+(-1))f(t) = 0 f(t) = 0$

(VS5) $(1\, g)(t) = 1\cdot g(t) = 1\cdot g(-t) = g(t)$.

(VS6) $(ab)\, g(t) = a(bg)t = a(bg(t))$

(VS7) $a[(f+g)(t)] = a[f(t) + g(t)] = af(t) + a\, g(t)$.

(VS8) $(a+b)(f(t)) = af(t) + bf(t)$.

Hence V is a vector space.

$\boxed{1.2.13}$

$$V: \left|\begin{array}{l} (a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2) \\ c(a_1, a_2) = (ca_1, a_2). \end{array}\right.$$