If $T^t f_1 = af_1 + cf_2$ then applying both sides to (x,y) gives $3x+2y = ax+cy \Rightarrow a=3$ and $c=2$

Similarly, $(T^t f_2)(x,y) = x$
$= 1.f_1 + 0.f_2$

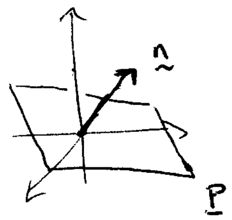
and so $[T^t]_{\beta^*}^{\beta^*} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$

(e) $T(1,0) = (3,1)$ and $T(0,1) = (2,0)$
$= 3.(1,0) + 1.(0,1)$ $= 2.(1,0) + 0.(0,1)$

So $[T]_\beta^\beta = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$

Thus $[T^t]_{\beta^*}^{\beta^*} = ([T]_\beta^\beta)^T$, as we expect from Theorem 2.25.

§2.6 #8



Given a plane $\underline{P} \subseteq \mathbb{R}^3$ which passes through the origin we can find a vector $\underset{\sim}{n} = (a,b,c)$ which is perpendicular to $\underline{P}$. Thus $\underset{\sim}{x} = (x,y,z)$ is in $\underline{P}$ if and only if $\underset{\sim}{x} . \underset{\sim}{n} = 0$.

In other words, $\underline{P} = \{(x,y,z) \in \mathbb{R}^3 : ax+by+cz = 0\}$
$= N(f)$

where $f(x,y,z) = ax+by+cz$. It is easy to check that f is linear, i.e. that $f \in (\mathbb{R}^3)^*$.

[Note that our choice of f is not unique: λf would also do, for $\lambda \neq 0$, $\lambda \in \mathbb{R}$]