§2.6 #4 Since V^* is 3-dimensional, to show that $\{f_1, f_2, f_3\}$ form a basis for V^* it suffices to show that they are linearly independent. Suppose that

$$af_1 + bf_2 + cf_3 = 0 \quad \cdots\cdots\cdots \quad (1)$$

Applying (1) to the vector $(1,0,0)$ we find that

$$a.1 + b.1 + c.0 = 0$$

Applying (1) to the vector $(0,1,0)$ we find that

$$a.(-2) + b.1 + c.1 = 0$$

Applying (1) to the vector $(0,0,1)$ we find that

$$a.0 + b.1 + c.(-3) = 0$$

So

$$a + b = 0 \quad \cdots \quad (2)$$
$$-2a + b + c = 0 \quad \cdots \quad (3)$$
$$b - 3c = 0 \quad \cdots \quad (4)$$

Substituting (2) into (3) and (4) we find that $-3a + c = 0$ and $a + 3c = 0$

and therefore that $a = c = 0$. From (2) we conclude that $b = 0$ also.

Thus $\{f_1, f_2, f_3\}$ are LI, and hence a basis for V^*.

Suppose that $\{x_1, x_2, x_3\}$ is a basis for V which is dual to $\{f_1, f_2, f_3\}$ and that $x_1 = (a,b,c)$. Since

$$f_1(x_1) = 1 \qquad f_2(x_1) = 0 \qquad \text{and} \qquad f_3(x_1) = 0$$

we know that