CORRECTIONS TO THE SOLUTIONS TO THE FIRST HOMEWORK

There are a couple of typos in the solutions to the first homework. Corrections are as follows.

- 1.2.12 (VS 4) We know from (VS 3) that the zero function is defined by $0(t) = 0$ for all $t \in \mathbb{R}$; define the function g (which is going to be $-f$) by setting $g(t) = -f(t)$ for all $t \in \mathbb{R}$. Then we want to show that $f + g = 0$, in other words that $f(t) + g(t) = 0(t)$ for all $t \in \mathbb{R}$. But $f(t) + g(t) = f(t) - f(t) = 0$ and $0(t) = 0$ too, so (VS4) holds.
- 1.2.12 (VS 8) we know that the function af, where $a \in \mathbb{R}$ and $f \in V$, is defined by $(af)(t) = af(t)$ for all $t \in \mathbb{R}$. We want to show that $(a + b)f = af + bf$ for all $a, b \in \mathbb{R}$ and $f \in V$, so we need to show that $((a + b)f)(t) = (af + bf)(t)$ for all $t \in \mathbb{R}$. But $((a + b)f)(t) = (a + b)f(t) = af(t) + bf(t)$ and $(af + bf)(t) = (af)(t) + (bf)(t) = af(t) + bf(t)$, so (VS 8) holds.
- 1.2.16 The question asks you to prove that $M_{m,n}(\mathbb{R})$ is a vector space over $\mathbb{Q}$, so you *cannot* use the example in class — this will show that $M_{m,n}(\mathbb{Q})$ is a vector space over $\mathbb{Q}$, but that wasn't what the question asked. You need to check that addition and scalar multiplication are defined (which follows from the facts that $M_{m,n}(\mathbb{R})$ is a vector space over $\mathbb{R}$ and that rational numbers are real numbers) and check the axioms. These also all follow from the facts that $M_{m,n}(\mathbb{R})$ is a vector space over $\mathbb{R}$ and that rational numbers are real numbers.
- 1.3.10 "closure under multiplication" should read "closure under addition".
- 1.2.26 The definition of W_2 should have "$i \leq j$" not "$i < j$", and consequently the remark "no-one says it can't have zeros on the diagonal" should read "it must have zeros on the diagonal".
- 1.4.10 "$aM_1 + aM_1 + aM_1$" should read "$aM_1 + bM_1 + cM_1$".