

SUMMARY OF AXIOMS

1. AXIOMS FOR A VECTOR SPACE

A *vector space* V over a *field* \mathbb{F} is a set V together with two operations, called addition and scalar multiplication, such that

- for each x and y in V there is a unique element $x + y$ in V .
- for each x in V and each a in \mathbb{F} there is a unique element ax in V .

and

- (VS1) for all x and y in V , $x + y = y + x$.
- (VS2) for all x , y and z in V , $(x + y) + z = x + (y + z)$.
- (VS3) there is an element 0 of V such that $x + 0 = x$ for all x in V .
- (VS4) for each x in V there is an element y in V such that $x + y = 0$.
- (VS5) for each x in V , $1x = x$.
- (VS6) for each a and b in \mathbb{F} , and each x in V , $(ab)x = a(bx)$.
- (VS7) for all x and y in V and all a in \mathbb{F} , $a(x + y) = ax + ay$.
- (VS8) for all a and b in \mathbb{F} and all x in V , $(a + b)x = ax + bx$.

2. AXIOMS FOR A FIELD

A *field* \mathbb{F} is a set together with two operations, called addition and multiplication, such that

- for each a and b in \mathbb{F} there is a unique element $a + b$ in \mathbb{F}
- for each a and b in \mathbb{F} there is a unique element ab in \mathbb{F}

and

- (F1) for all a and b in \mathbb{F} , $a + b = b + a$ and $ab = ba$.
- (F2) for all a , b and c in \mathbb{F} , $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.
- (F3) there are distinct elements 0 and 1 in \mathbb{F} such that for all a in \mathbb{F} , $0 + a = a$ and $1a = a$.
- (F4) for each element a in \mathbb{F} and each non-zero element b in \mathbb{F} there are elements c and d in \mathbb{F} such that $a + c = 0$ and $bd = 1$.
- (F5) for all a , b and c in \mathbb{F} , $a(b + c) = ab + ac$.