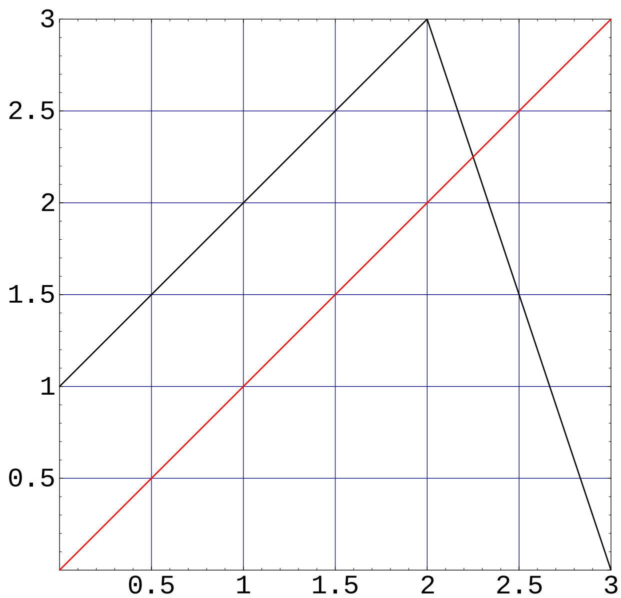
Problem Set 5

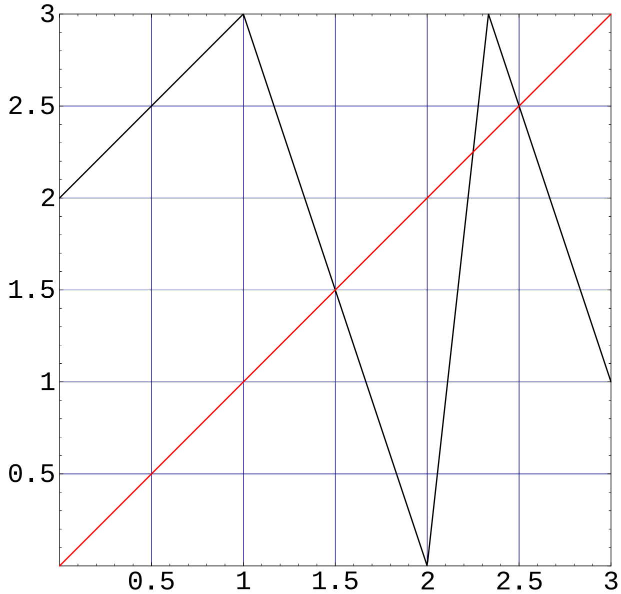
■ Exercise 11.4

```
In[1]:= F[x_] := Which[
          x ≤ 2, x + 1,
          x ≥ 2, -3 x + 9]

In[2]:= NestList[F, 0, 4]

Out[2]= {0, 1, 2, 3, 0}
```

So there's an obvious four−cycle. Hence there must be cycles of periods two and one, which we can see on the graphs.

```
In[3]:= Table[Plot[{Nest[F, x, j], x}, {x, 0, 3},
          PlotStyle -> {{}, RGBColor[1, 0, 0]},
         GridLines -> Automatic,
          PlotRange -> {{0, 3}, {0, 3}},
         Frame -> True, Axes -> False,
        AspectRatio -> 1], {j, 4}]
```




3
2.5
2
1.5
1
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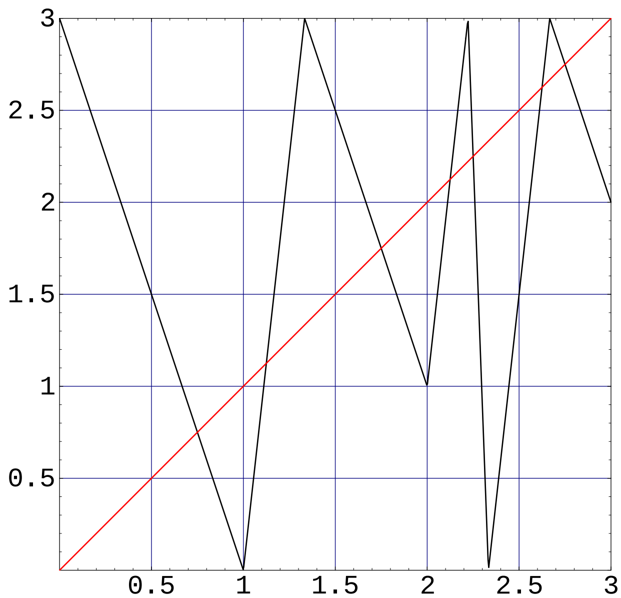
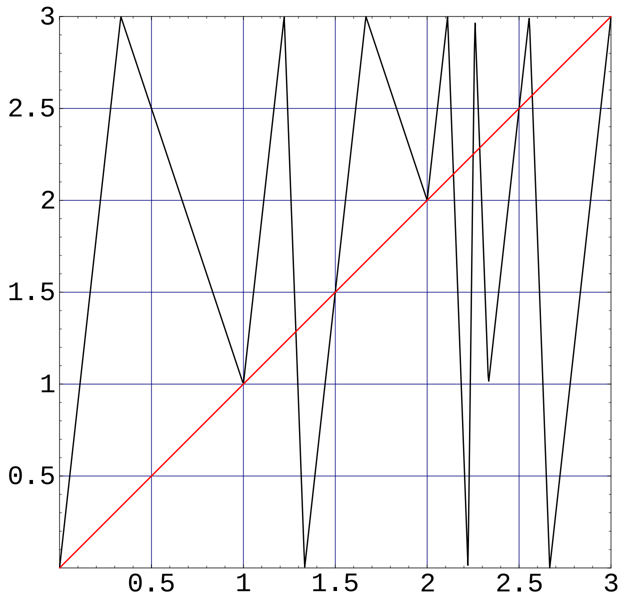
0.5 1 1.5 2 2.5 3


3
2.5
2
1.5
1
0.5
0.5 1 1.5 2 2.5 3




```
Out[3]= {- Graphics -, - Graphics -, - Graphics -, - Graphics -}

In[4]:= NestList[F, 3 / 2, 2]
```

Out[4]= $\{\frac{3}{2}, \frac{5}{2}, \frac{3}{2}\}$

```
In[5]:= F[9 / 4]
```

Out[5]= $\frac{9}{4}$

Here's a guess, which turns out to be right.

```
In[6]:= NestList[F, 11 / 4, 3]
```

Out[6]= $\{\frac{11}{4}, \frac{3}{4}, \frac{7}{4}, \frac{11}{4}\}$

So F has a three−cycle, hence periodic points of all other periods.

On the other hand, define

```
In[7]:= G[x_] := Which[
          x ≤ 1, -x + 3,
          x ≤ 2, -2 x + 4,
         x ≤ 3, x - 2]

In[8]:= NestList[G, 0, 4]

Out[8]= {0, 3, 1, 2, 0}

In[9]:= Table[Plot[{Nest[G, x, j], x}, {x, 0, 3},
          PlotStyle -> {{}, RGBColor[1, 0, 0]},
         GridLines -> Automatic,
          PlotRange -> {{0, 3}, {0, 3}},
         Frame -> True, Axes -> False,
        AspectRatio -> 1], {j, 8}]
```

So we have found the fixed point and two−cycle. If G has periodic points of any other (prime) periods, it must have one of prime period 8. But this is not true, as can be seen from the graph. The only 8−periodic points to be accounted for are of lower prime period.

■ Exercise 11.6

```
In[12]:= H[x_] := Which[
            x ≤ 2, 3 x + 1,
            x ≤ 4, -x + 9,
           x ≤ 5, -2 x + 13,
           x ≤ 7, -x + 8]
```

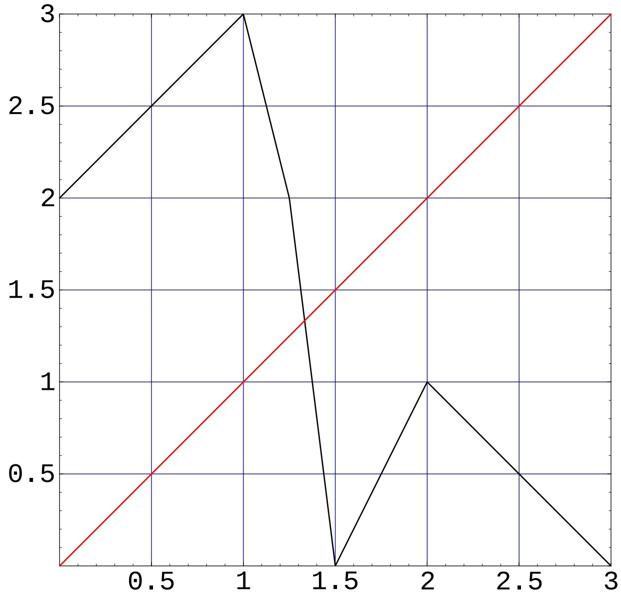
```
Out[9]= {- Graphics -, - Graphics -, - Graphics -, - Graphics -, - Graphics -, - Graphics -,
         - Graphics -, - Graphics -}

In[10]:= G[4 / 3]
```

Out[10]= $\frac{4}{3}$

```
In[11]:= NestList[G, 1 / 2, 2]
```

Out[11]= $\left\{\frac{1}{2}, \frac{5}{2}, \frac{1}{2}\right\}$


3
2.5
2
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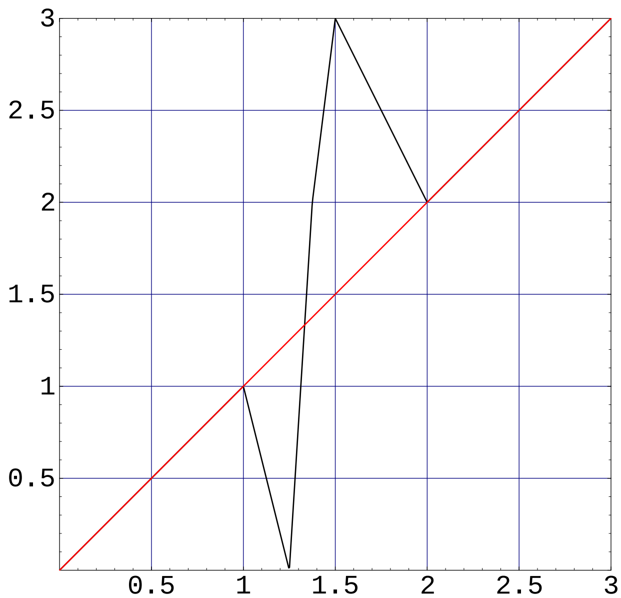
0.5 1 1.5 2 2.5 3


3
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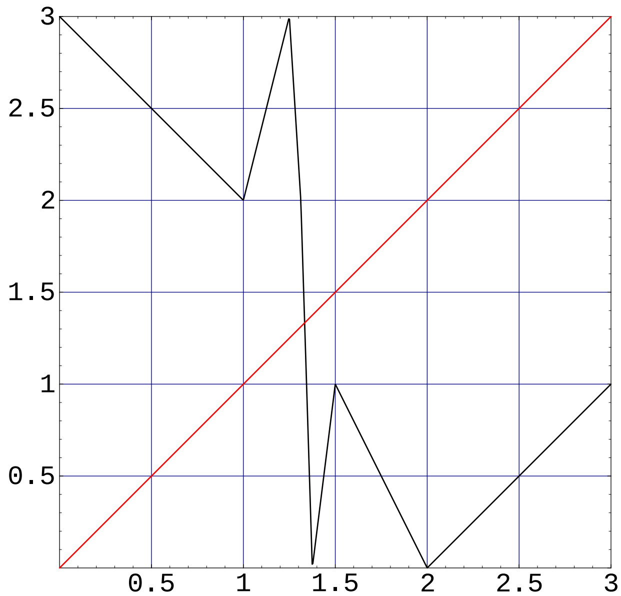
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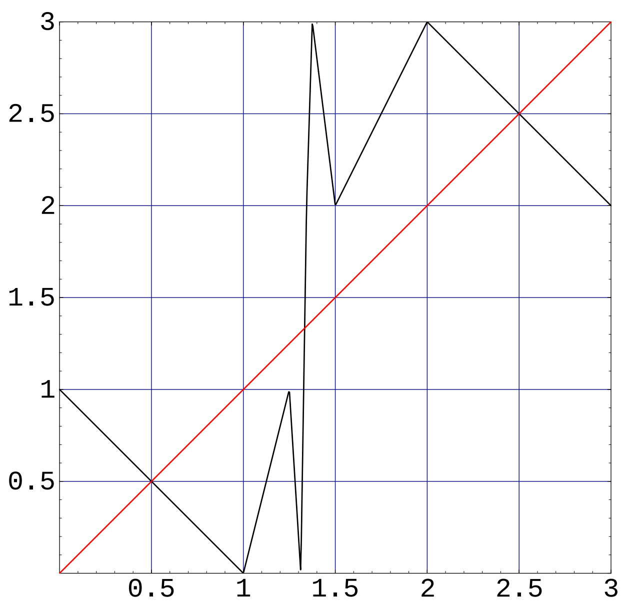
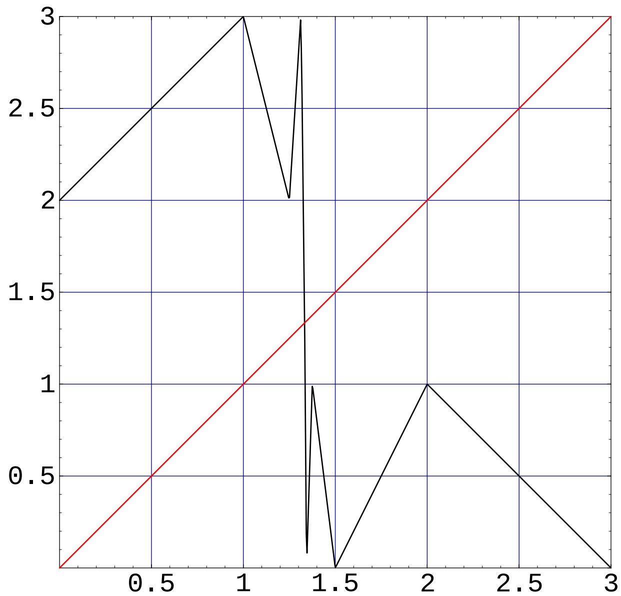
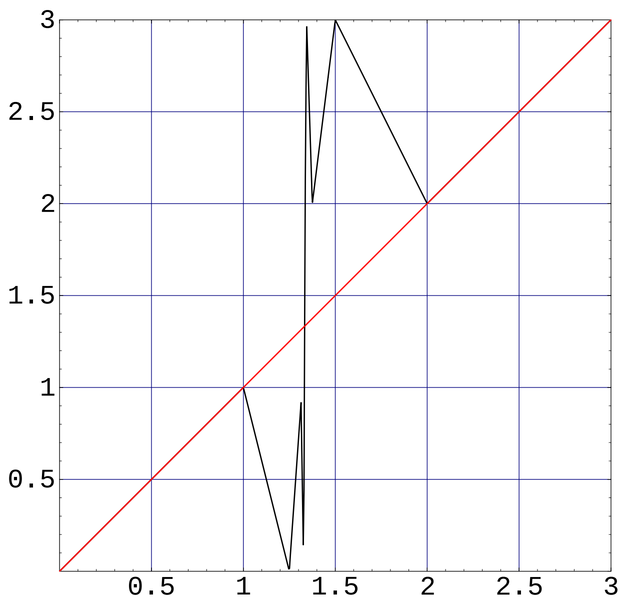
0.5 1 1.5 2 2.5 3


3
2.5
2
1.5
1
0.5
0.5 1 1.5 2 2.5 3






```
Out[9]= {- Graphics -, - Graphics -, - Graphics -, - Graphics -, - Graphics -, - Graphics -,
         - Graphics -, - Graphics -}
```

In[10]:= **G[4 / 3]**

Out[10]= $\frac{4}{3}$

In[11]:= **NestList[G, 1 / 2, 2]**

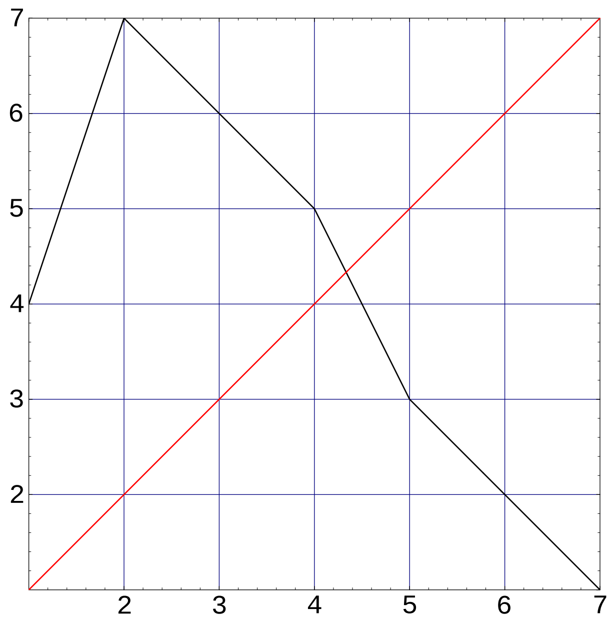
Out[11]= $\left\{\frac{1}{2}, \frac{5}{2}, \frac{1}{2}\right\}$

So we have found the fixed point and two−cycle. If G has periodic points of any other (prime) periods, it must have one of prime period 8. But this is not true, as can be seen from the graph. The only 8−periodic points to be accounted for are of lower prime period.

■ Exercise 11.6

```
In[12]:= H[x_] := Which[
           x ≤ 2, 3 x + 1,
           x ≤ 4, -x + 9,
          x ≤ 5, -2 x + 13,
          x ≤ 7, -x + 8]
```

```
In[13]:= Plot[{H[x], x}, {x, 1, 7},
           PlotStyle -> {{}, RGBColor[1, 0, 0]},
          GridLines -> Automatic,
           PlotRange -> {{1, 7}, {1, 7}},
          Frame -> True, Axes -> False,
         AspectRatio -> 1]
```



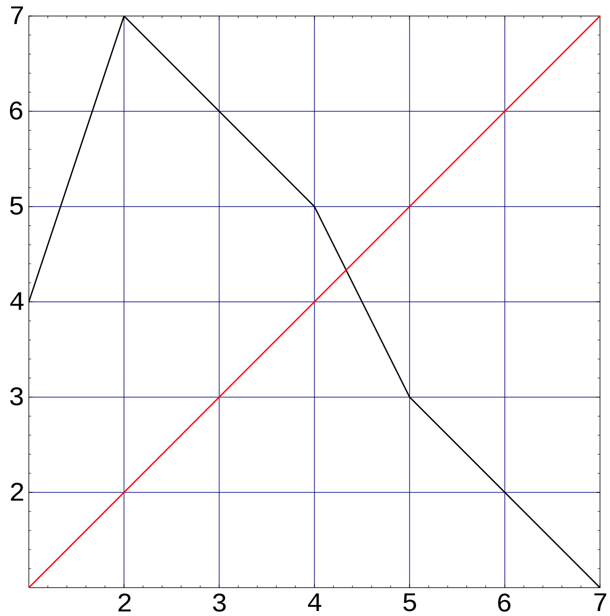

```
Out[13]= - Graphics -
```

```
In[14]:= NestList[H, 1, 7]
```

```
Out[14]= {1, 4, 5, 3, 6, 2, 7, 1}
```

So there is a seven−cycle.

```
In[15]:= Show[GraphicsArray[
          Table[Plot[{Nest[H, x, 2 i + j], x}, {x, 1, 7},
           PlotStyle -> {{}, RGBColor[1, 0, 0]},
          GridLines -> Automatic,
           PlotRange -> {{1, 7}, {1, 7}},
          Frame -> True, Axes -> False,
         AspectRatio -> 1], {i, 0, 3}, {j, 1, 2}]]]
```



7
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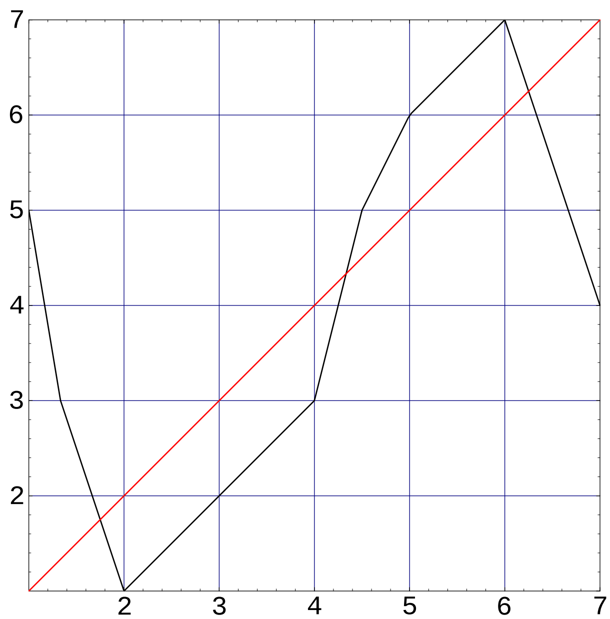

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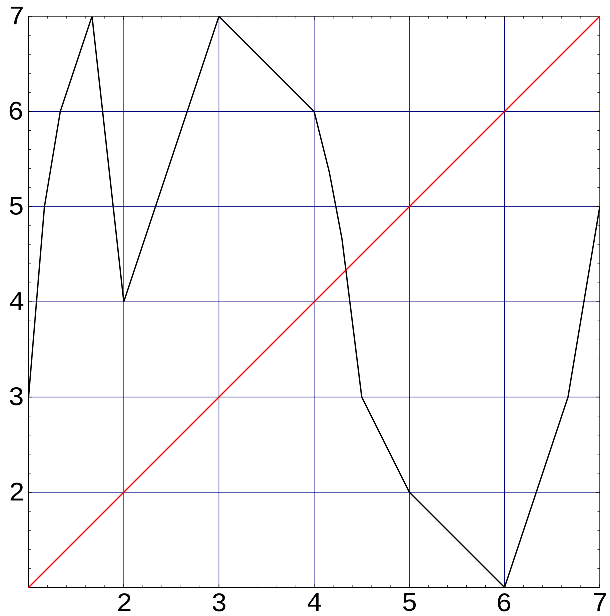
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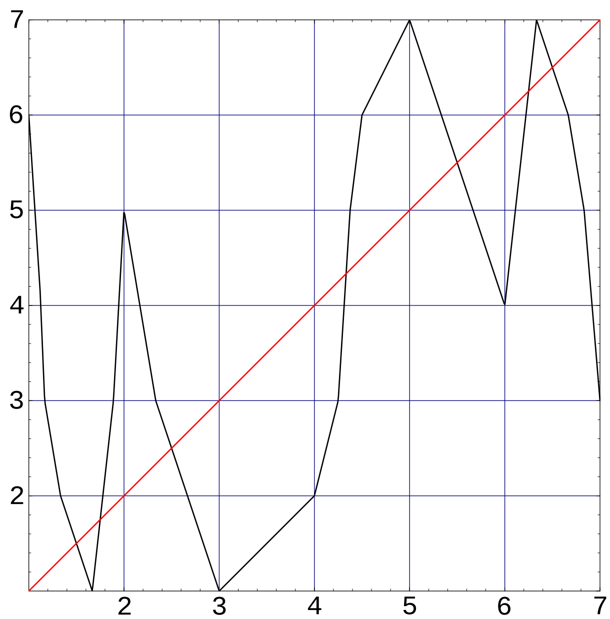
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