MATH 118
PROBLEM SET 3

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Exercise 1. (a) *Suppose* $f\colon \mathbb{R} \to \mathbb{R}$ *is continuous and* $f(x_0) > 0$. *Show that* $f(x) > 0$ *in a neighborhood of* x_0.

(b) *Explain how and why we used this result in one of our proofs.*

Exercise 2. *Let* X *be a complete metric space.*

(a) *Show that if* $T\colon X \to X$ *is a contraction, then* T^n *is, too.*

(b) *Give an example of a continuous* $T\colon X \to X$ *that is not a contraction but* T^n *is a contraction for some* n.

Exercise 3. (a) *See by direct calculation where the equation* $x^2 + x + 3u + 1 = 0$ *defines* x *implicitly as a function of* u.

(b) *Explain whether your answer agrees with the Implicit Function Theorem's.*

Exercise 4. *Suppose* T *maps a complete metric space* (X, d) *to itself and satisfies* $d(Tx, Ty) \leq d(x, y)$ *for all* x *and* y *in a closed ball* $Y = \{\, x \mid d(x, x_0) \leq r \,\}$. *Give a bound on* $d(x_0, Tx_0)$ *that allows you to state a prove a fixed point theorem for* T.

Exercise 5. (a) *One non-Newtonian method for solving* $f(x) = x^3 + x - 1 = 0$ *might be to iterate* $g(x) = (1 + x^2)^{-1}$. *Why? Try a few steps beginning with* $x_0 = 1$. *Can you estimate the errors, i.e., the distance from successive iterates from a solution?*

(b) *Show that another method would be to iterate* $g(x) = x^{1/2}(1 + x^2)^{-1/2}$. *Chose* $x_0 = 1$, *calculate a few steps, and explain what is happening.*

Also Continue your group computer projects by exploring orbits of $L_\mu(x) = \mu x(1 - x)$.

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