MATH 118 : SPRING 1999
CHALLENGE PROBLEMS

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Problem 1. Investigate the converse of Šarkovskii's theorem. For example, show that $5 \not\triangleright 3$.

Problem 2. Consider the shift σ restricted to the set $\Sigma' \subset \Sigma_2$ consisting of all sequences with no repeated zeros ($s_i = 0 \implies s_{i+1} = 1$). Show this "subshift" is chaotic. Can you find an expression for $\#\,\mathrm{Per}_k$ in terms of $\#\,\mathrm{Per}_{k-1}$ and $\#\,\mathrm{Per}_{k-2}$?

Problem 3. Show that Newton's method for solving $f(x) = (x - a)(x - a)$ is conjugate to $g(y) = y^2$ by a map $\varphi(y) = \frac{by-a}{y-1}$. Find φ^{-1} and interpret (consider ∞ as just another point). Use this to completely describe the Newton dynamics from any initial point.

Problem 4. Find an example of a map T on a complete metric space such that T^m is a contraction but T is not. Must T have a fixed point? Find an example of a contraction on an incomplete metric space that has no fixed point.

Problem 5. Explain why μ must be large to prove that the logistic map L_μ is chaotic. Show that the "nonwandering set" equals the Cantor set Λ we found for μ sufficiently large.

Problem 6. Investigate the bifurcations of $S_\lambda(x) = \lambda \sin x$.

Problem 7. Show that $L_4(x) = 4x(1 - x)$ is chaotic.

Problem 8. Analyze the behavior of the family $f_c(x) = -(1+x)x - x^2 - 2x^3$ near $x^* = 0$ and $c^* = 0$.

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