3/21 (Start of Spring!) QUIZ NR 7 **Math118, O.Knill**

Name:

1) Which of the following formulations is the full content of the Hedlund-Lyndon-Curtis theorem? (only one answer is correct).

a) Any map T on $X = \{0,1\}^Z$ which is continuous and commutes with the shift is of the form $T(x)_n = \phi(x_{n-1}, x_n, x_{n+1})$.

b) A cellular automaton is a continuous map on $X = \{0,1\}^Z$.

c) A shift commuting, continuous map on $X = \{0,1\}^Z$ is a cellular automaton.

d) Any continuous map on $X = \{0,1\}$ has the property that the n'th entry of $T(x)$ depends only on finitely many neigbors.

2) True or False?

a) There exists a cellular automaton T such that the set of periodic orbits is dense.

a) There exists a cellular automaton T such that the set of periodic orbits of period 11 are dense.

b) There exists a cellular automaton T such that $\{T^n(x), n = 1, 2, ...\}$ covers the entire set $X = \{0,1\}^Z$.

c) There exists a cellular automaton T, such that $\{T^n(x), n = 1, 2, ...,\}$ is dense in X.

3) We have $T(x)_n = x_{n+1} + x_{n-1} + x_n \mod(2)$. What is the image of the sequence $x = (..., 1, 0, 1, 0, 1, 0, 1, ...)$?

a) (...,1,1,1,1,1,1,1,1,1,1,1,...)

b) (...,0,0,0,0,0,0,0,0,0,0,0,...)

c) (...,1,0,1,0,1,0,1,0,1,0,1....)

4) Assume a cellular automaton has the property that $T^3(x)$ is the shift. Which of the following statements are true?

a) T is chaotic in the sense of Devaney.

b) T^3 is chaotic in the sense of Devaney.

c) T^9 is chaotic in the sense of Devenay

5) If $x = (...1, 0, 1, 0, 1, 0, 1, 0, 1, 0, ...$ with $x_0 = 1$ and $y = (..., 0, 0, 0, 0, 0, ...)$, then the distance between these two points $d(x, y)$ is

a) 1
b) 1/2
c) 0
d) 2

6) True or false?

If $d(x, y) = 1/10$, then $d(\sigma(x), \sigma(y)) = 1/10$, where σ is the shift.

7) A lattice gas cellular automaton

a) conserves the total momentum of the particles

b) is used to simulate fluids

c) is used to simulate sand dynamics.

d) conserves the total angular momentum of the particles.

e) conserves the total energy of the particles.

8) What is a "glider" in the game of life (X, T)?

a) A configuration x which satisfies $T^n(x) = \sigma^m(x)$ for $n, m > 0$.

b) A configuration with finitely many living cells which satisfies $T^n(x) = \sigma^m(x)$ for $n, m > 0$.

c) A configuration which satisfies $T^n(x) = x$ for $n > 0$.

d) A fixed point of T.

9) Who is believed to have first come up with the notion of cellular automata?

a) Hedlund at Harvard

b) Wolfram at Caltech

b) Ulam and von Neuman at Los Alamos

10) If you allow the alphabet of a cellular automaton to become a continuum, then the corresponding dynamical system is called a

a) partial differential equation.

b) coupled map lattice.

c) map on an infinite dimensional space

d) an infinite system of coupled ordinary differential equations.

11) (5 points if correct) A one dimensional automaton maps x to y, where

x =	...	1	1	1	0	1	1	0	0	1	1	0	0	0	0	1	0	...
y =	...	1	0	0	1	1	0	0	0	1	0	0	1	1	0	0	1	...

What is the Wolfram number of this cellular automaton?