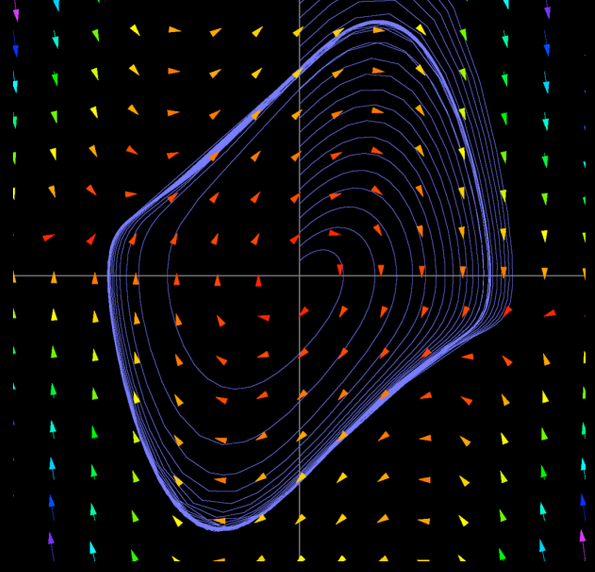
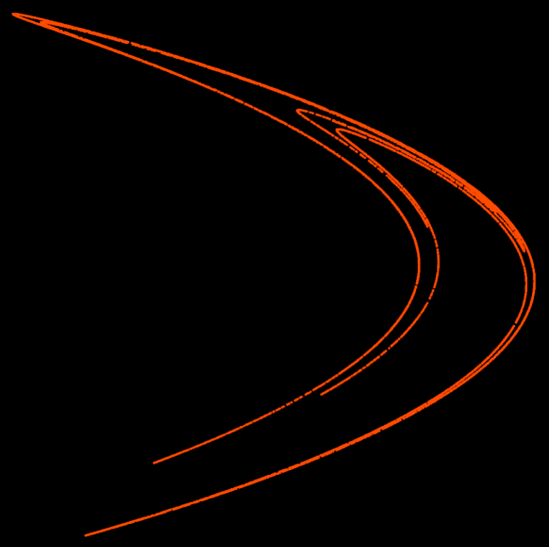
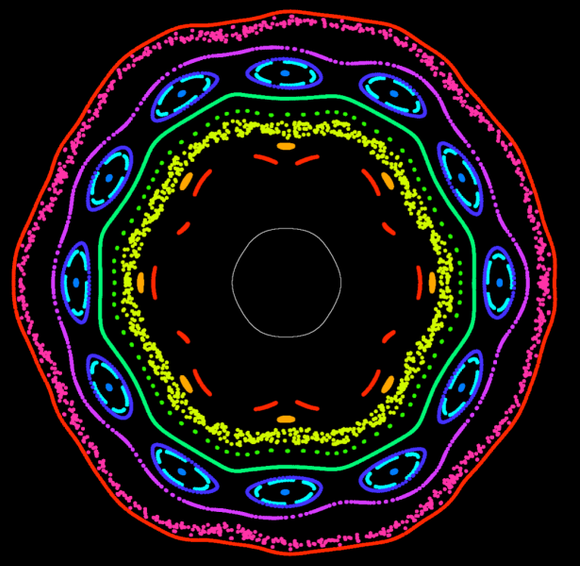


Math 118r Introduction



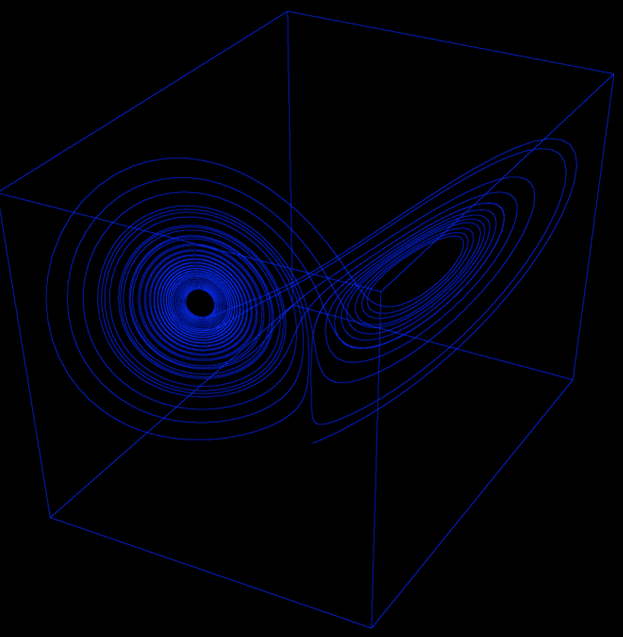


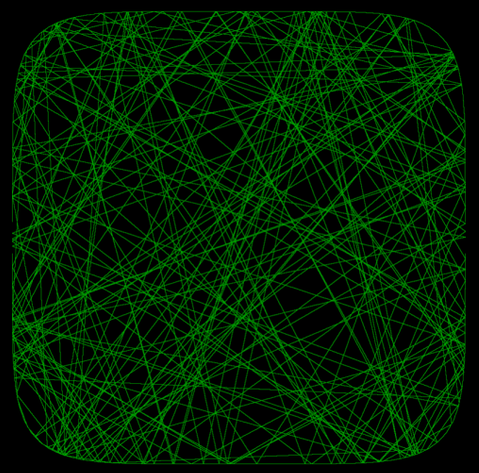
A: What is a dynamical system?

B: Relevance of predictability

C: Organization of the course.

D: Syllabus walk through





Oliver Knill, Feb. 2, 2004

About Math118r:

- one semester of linear algebra as well as one semester of multivariable calculus are required.
- while the book talks in the back about senior undergraduates or graduate students, this is course is accessible for freshmen and sophomores.
- We focus on examples with occasional disgression into generalities.

A: What is dynamics?

- Model a concrete situation
- Predict future state of a system
- Understand the limitations of models
- Solve mathematical problems in other fields like geometry or number theory
- Mathematicians: semigroup

Mathematical Fields

Algebra

Measure Theory

Analysis

Topology

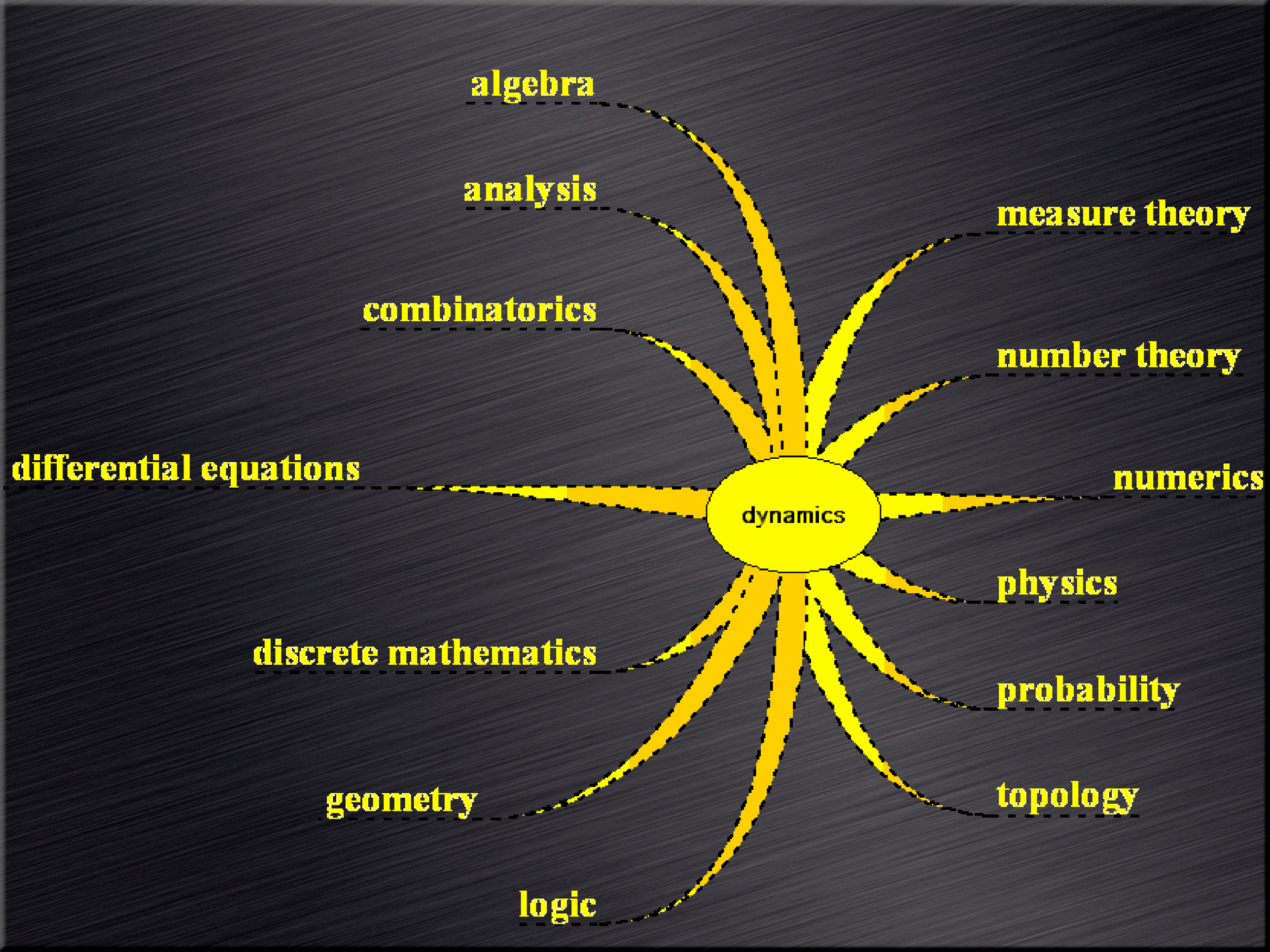
Probability

Geometry

Logic

Dynamics

Number Theory


algebra
analysis
combinatorics
differential equations
measure theory
number theory
numerics
dynamics
physics
discrete mathematics
probability
geometry
topology
logic

One of the main goals of dynamical system theory:

Predict the future of a specific system.

Let us look at some examples collected from news stories in the last few weeks (december 2004-january 2005)

Sybil Trelawney
(Harry Potter and the prisoner from Azkaban)

Mathematical definition:

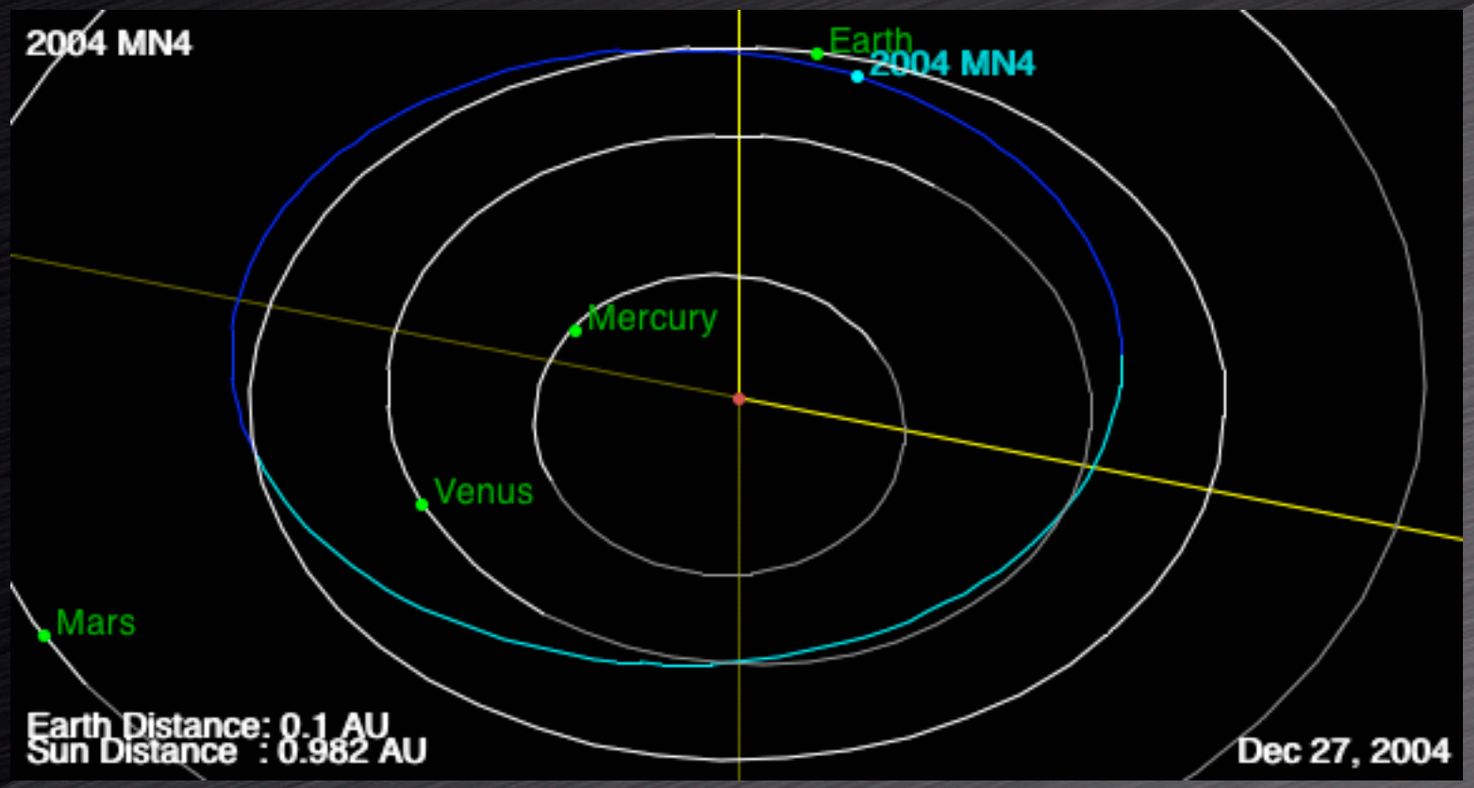
A semigroup is a set G with an operation *
which is associative (u*v)*w=u*(v*w)

A semigroup G acts on a set X,
if for each t in G, there is a
transformation T(t) on X such
that T(t*s)(x) = T(t) T(s)(x)

A dynamical system is a
semigroup G acting on a
set X.

While this is much
too abstract and
general for us
and just about
the contrary to
the approach we
use (focus on
examples)
it is useful to keep
this in mind.

Predictability I

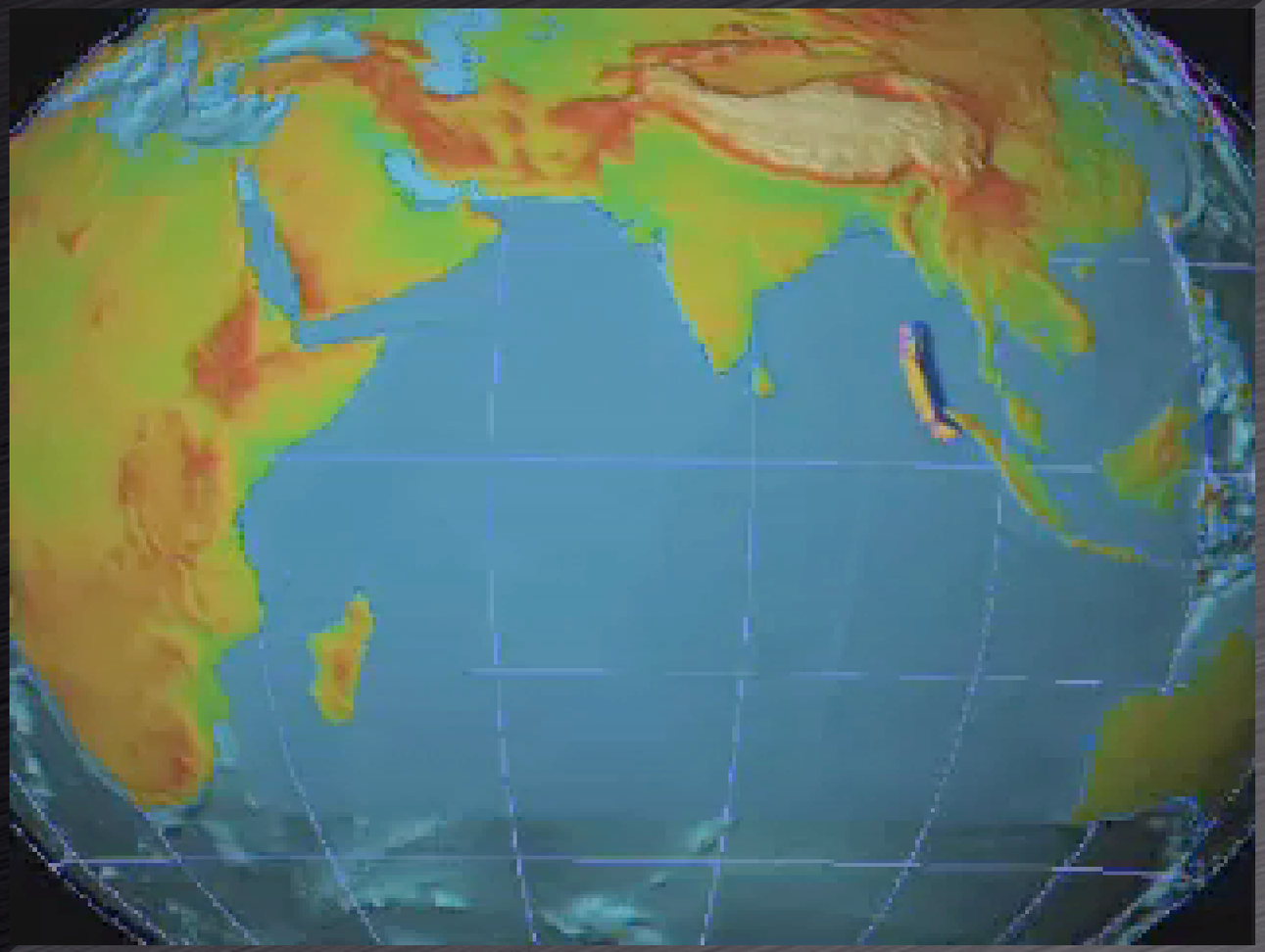


Asteroid 2004 MN4

The recently rediscovered Near Earth Asteroid 2004 MN4 may represent an impact hazard in 2029. This asteroid has an estimated diameter of 400 meters, and the nominal orbit solution results in a close approach to the Earth at 520,000 Km minimum distance on April 13, 2029.

However, given the current knowledge of the orbit, we cannot exclude that there could be an impact in that date. We have every expectation that further monitoring and further analysis on this object will entirely eliminate its potential hazard.

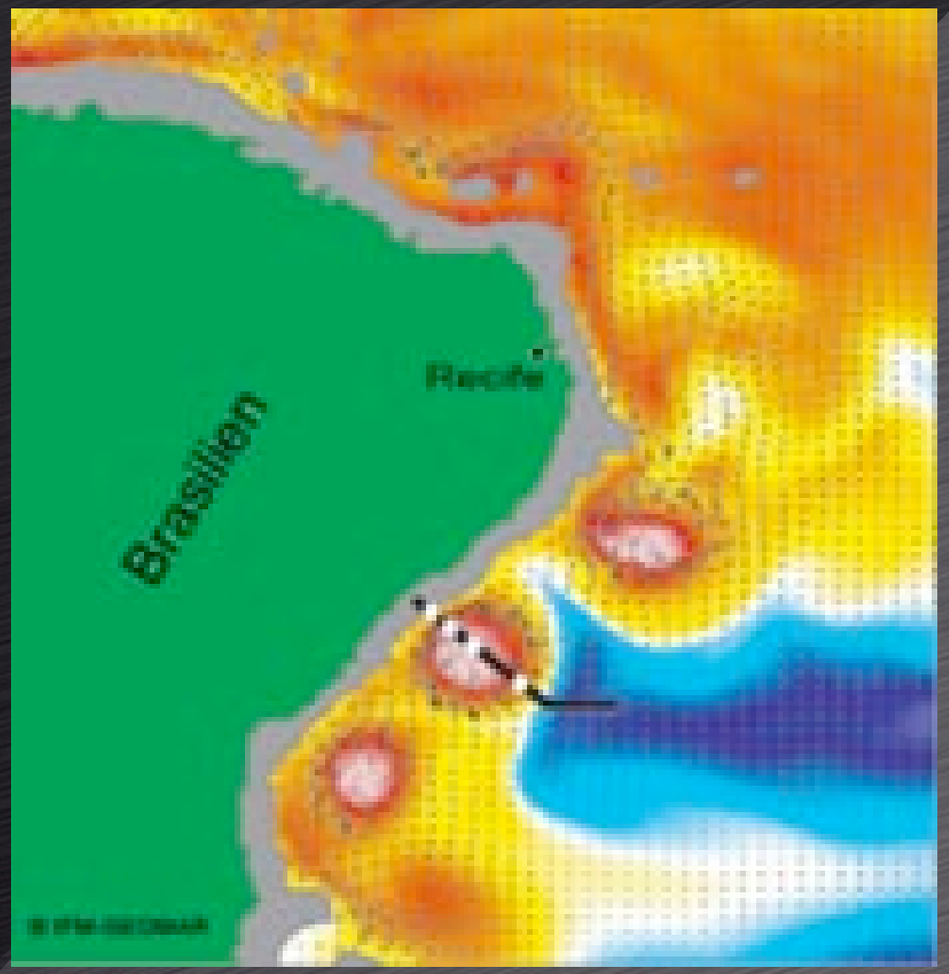
Predictability II



Simulation of wave dynamics in the Indian ocean was done only after the devastating tsunamy last December. It could have saved thousands of lives.

2 hours after the quake there was still no established model on what a tsunami might do in the Indian Ocean. 90 % happen in the Pacific. Vasily V. Titov in Seattle started computations. It was too late.

Predictability III



Recently, large vortex storms had been discovered near the South American continent. Understanding them could predict for example changes in the Gulf stream which would have huge climatic consequences.

Predictability IV

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230664709384460955058223172535940812848111745028410270193852110555964462294895493038196442881097566593344612847564823378678316527120190914564856692346034861045432664821339360726024914127372458700660631558817488152092096282925409171536436789259036001133053054882046652138414695194151160943305727036575959195309218611738193261179310511854807446237996274956735188575272489122793818301194912983367336244065664308602139494639522473719070217986094370277053921717629317675238467481846766940513200056812714526356082778577134275778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999837297804995105973173281609631859502445945534690830264252230825334468503526193118817101000313783875288658753320838142061717766914730359825349042875546873115956286388235378759375195778185778053217122680661300192787661119590921642019 9

Digits of Pi: does every digit occur with the same frequency?

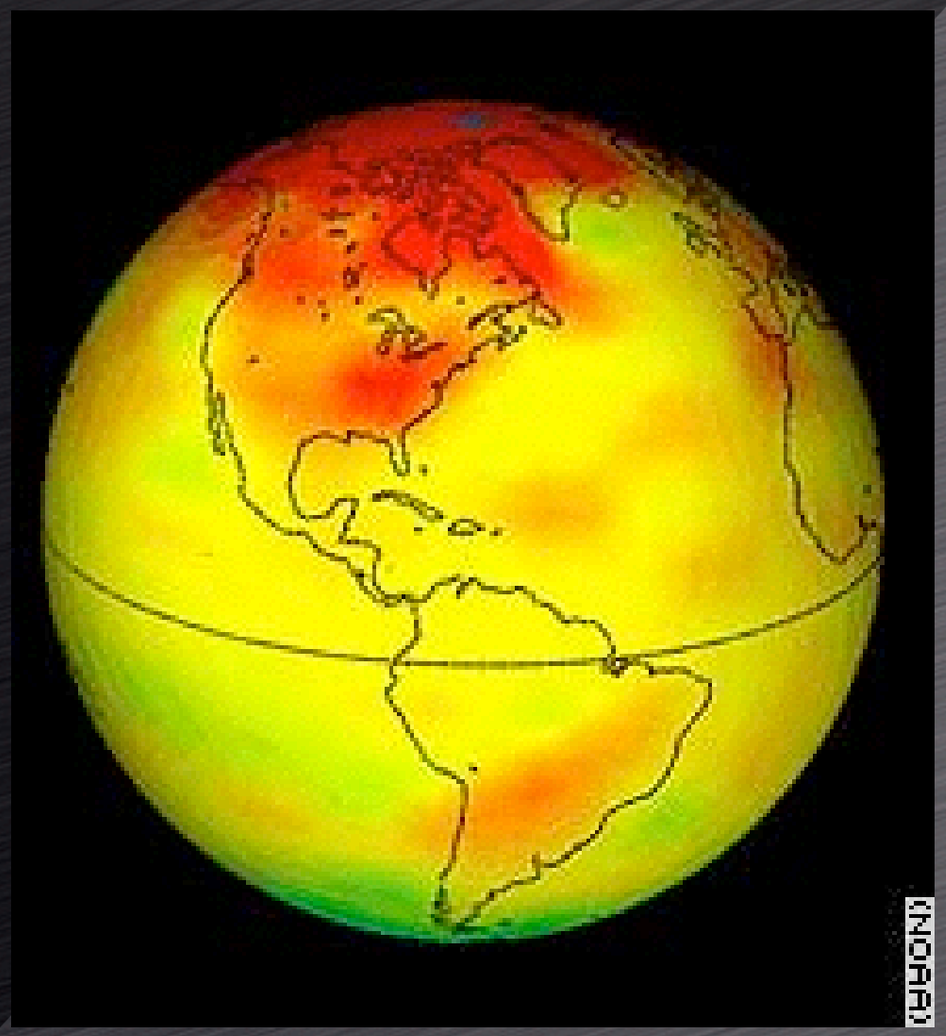
Predictability V

In January, a 100 mile long iceberg was found on collision course with a floating glacier.

Predictability VI

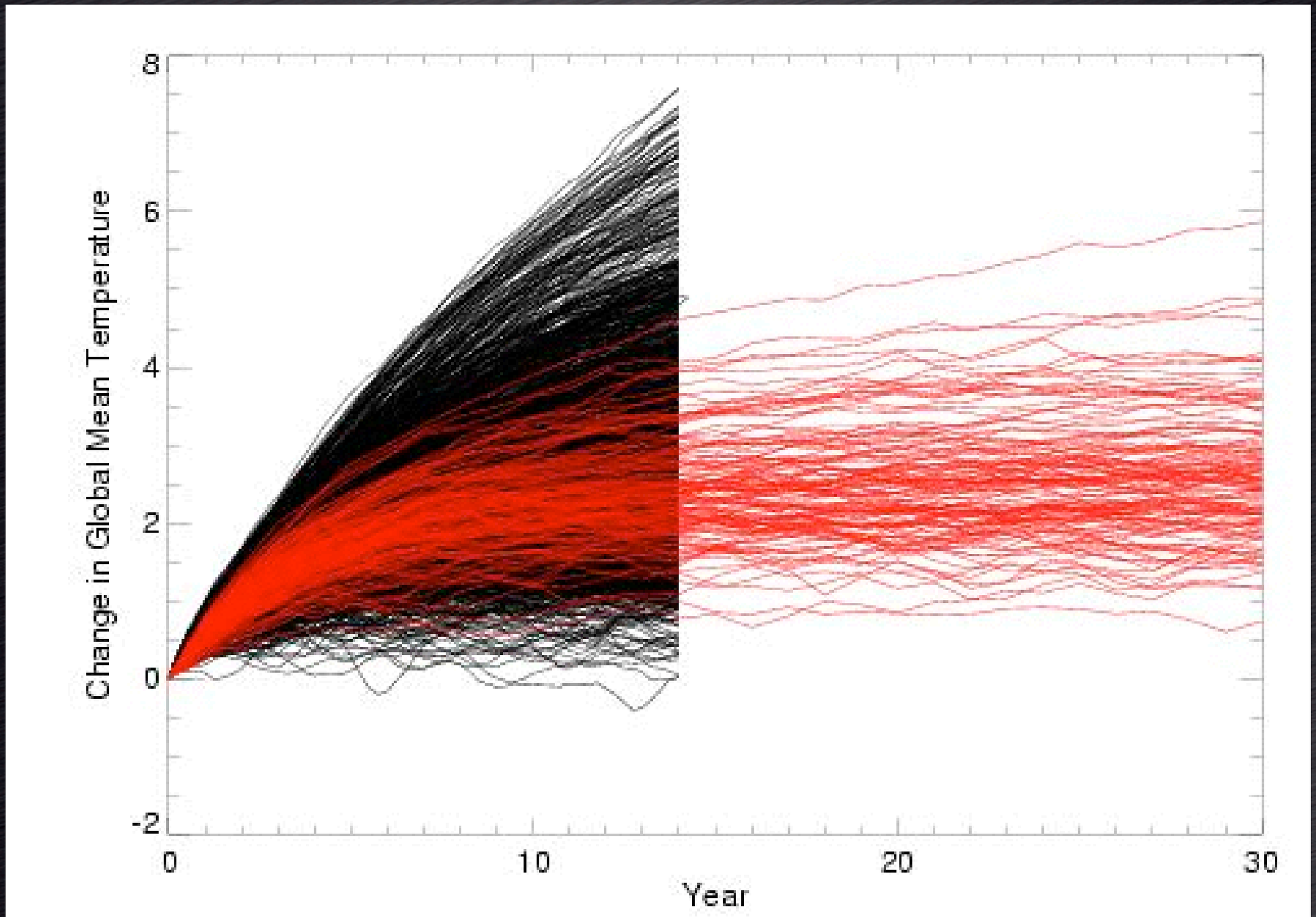
Using a cellular phone and a computer, a group of people were able to predict the Roulette numbers in London: they won1.3 Millon Pounds until they got arrested.

Predictability VII

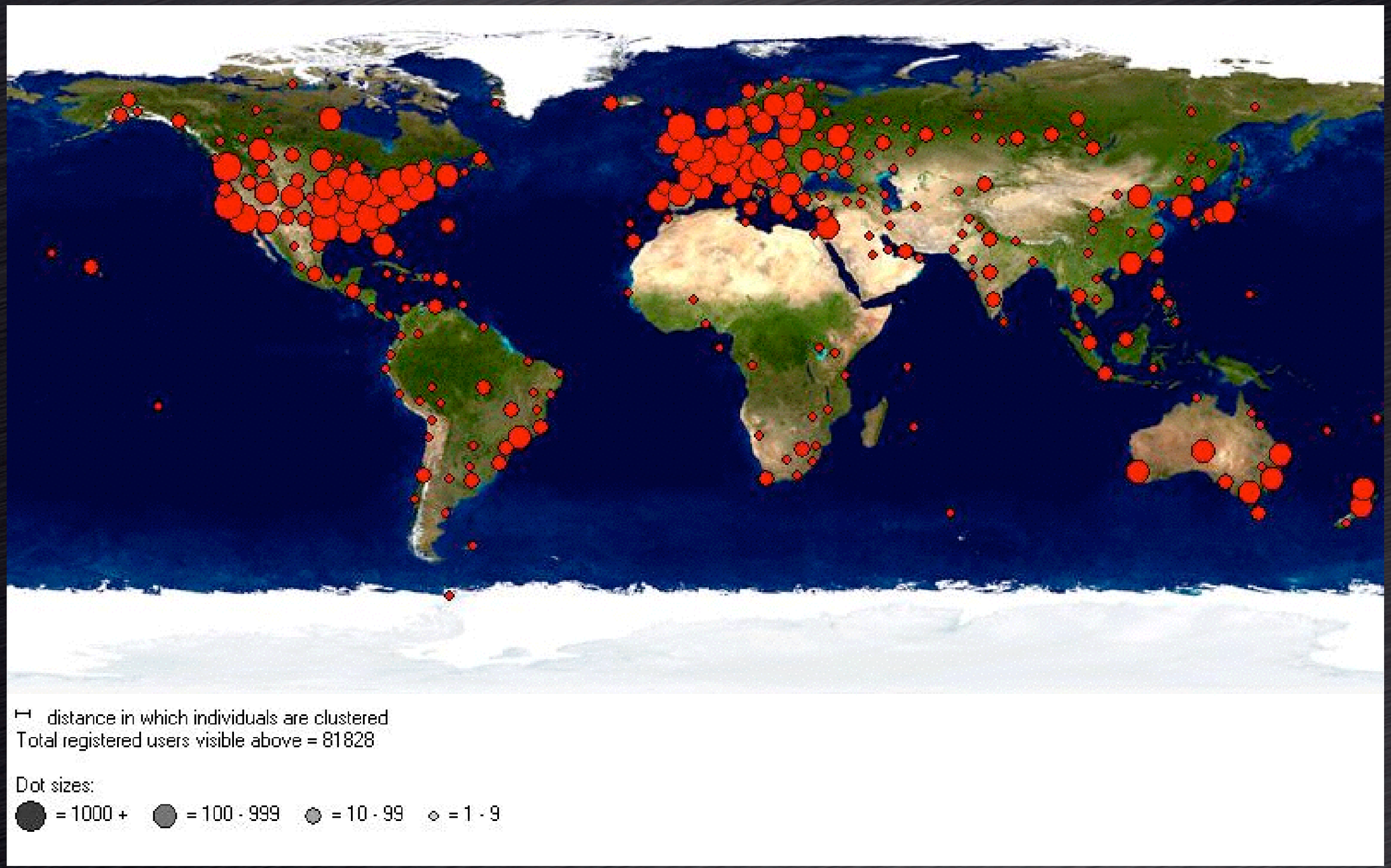


Does the earth heat up? Does This is an important question. Many computers around the world help with those computations using free CPU time. One of my computers helps too.

Here are some computation results (January 2005)



Using many home computers:



distance in which individuals are clustered
Total registered users visible above = 81828

Dot sizes:
= 1000 + = 100 - 999 = 10 - 99 = 1 - 9

News

Crichton best-seller stokes fire over **global warming**

Japan Today, Japan - 3 hours ago

WASHINGTON — Michael Crichton, author of "Jurassic Park," and his newest book cast doubt the danger of **global warming**, an issue dividing scientists and **...**

Global warming row over Crichton thriller Gulf Daily News

David B. Sandalow: Michael Crichton and **Global Warming** YubaNet (satire)

New novel turns tables on **global warming** theory Kansas City Star (subscription)

Miami Herald (subscription) - Indianapolis Star - all 6 related »

Discovery Channel

African Poor to Bear Brunt of **Global Warming** Crisis

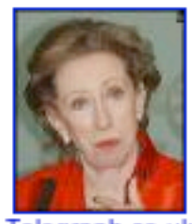
Reuters - 2 hours ago

EXETER (Reuters) - Africa's poor millions, already suffering grinding poverty and rampant disease, risk bearing the brunt of the **global warming** crisis unless **...**

Grim News Seen from Climate Meeting Discovery Channel

Study Bolsters Greenhouse Effect Theory, Solves Ice Age Mystery Science Daily (press release)

all 5 related »



Telegraph.co.uk

Global warming inevitable for decades to come, science conference **...**

Turkish Press, Turkey - 6 hours ago

EXETER, England (AFP) - A climate conference opened to renewed concern about the worsening threat of **global warming** and appeals from Britain to its ally, the **...**

Help tackle **global warming**: British call to US at climate **...** Hindustan Times

Scientists in parlay on **global warming** crisis ABC Online

Seeking **global warming's** danger zone International Herald Tribune

Scotsman - ABC Online - all 79 related »

Speaking tirelessly of **global warming**

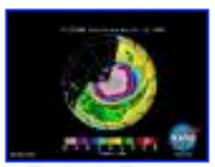
Ha'aretz, Israel - 2 hours ago

At the beginning of his speech at the Davos Conference last week, British Prime Minister Tony Blair cited **global warming** as one of the five most important **...**

Nuclear power given glowing approval to help curb **global warming**

Sydney Morning Herald (subscription), Australia - 13 hours ago

By Stephanie Peatling. Nuclear energy should be considered as part of the solution to **global warming**, one of Australia's most senior scientists says. **...**

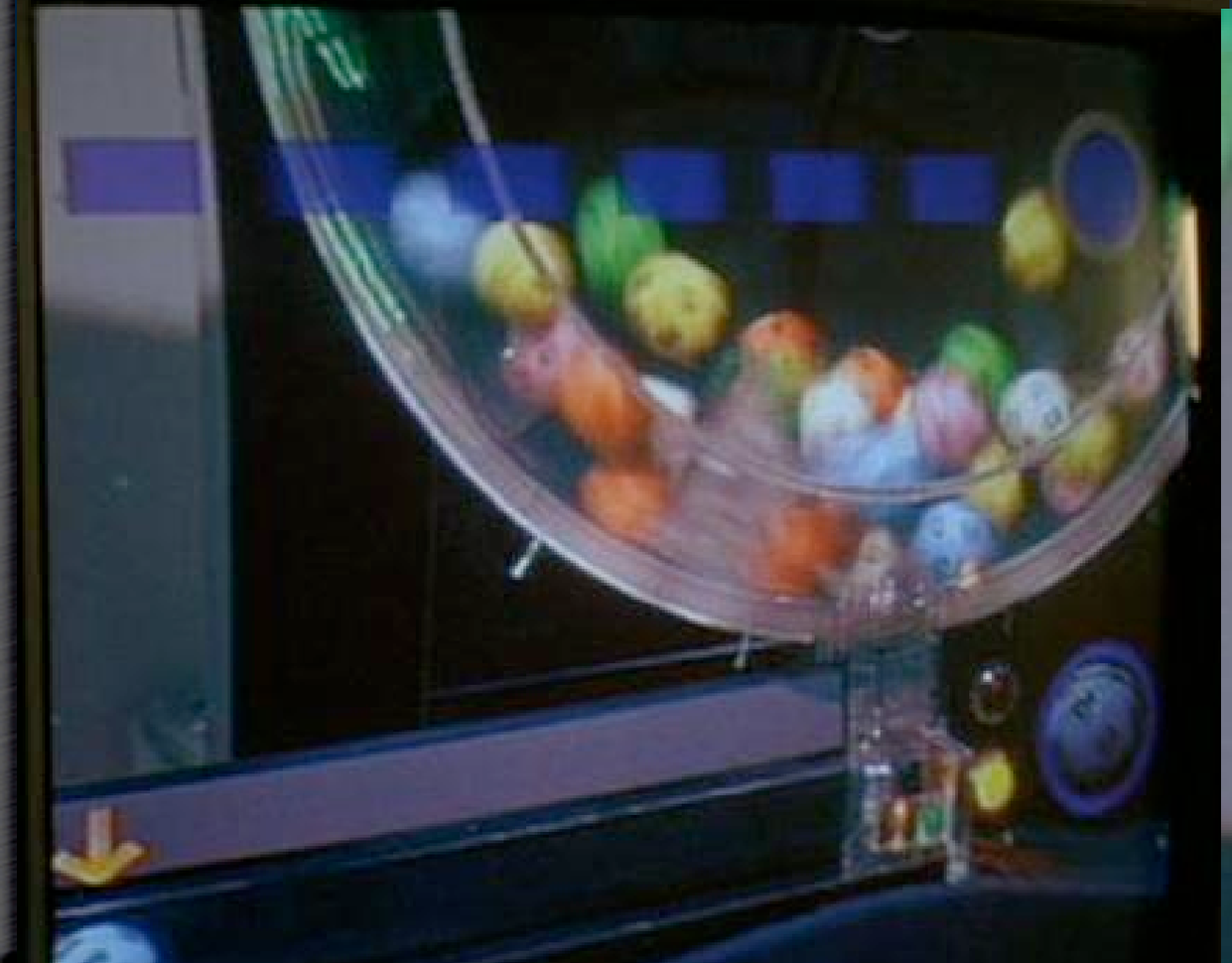


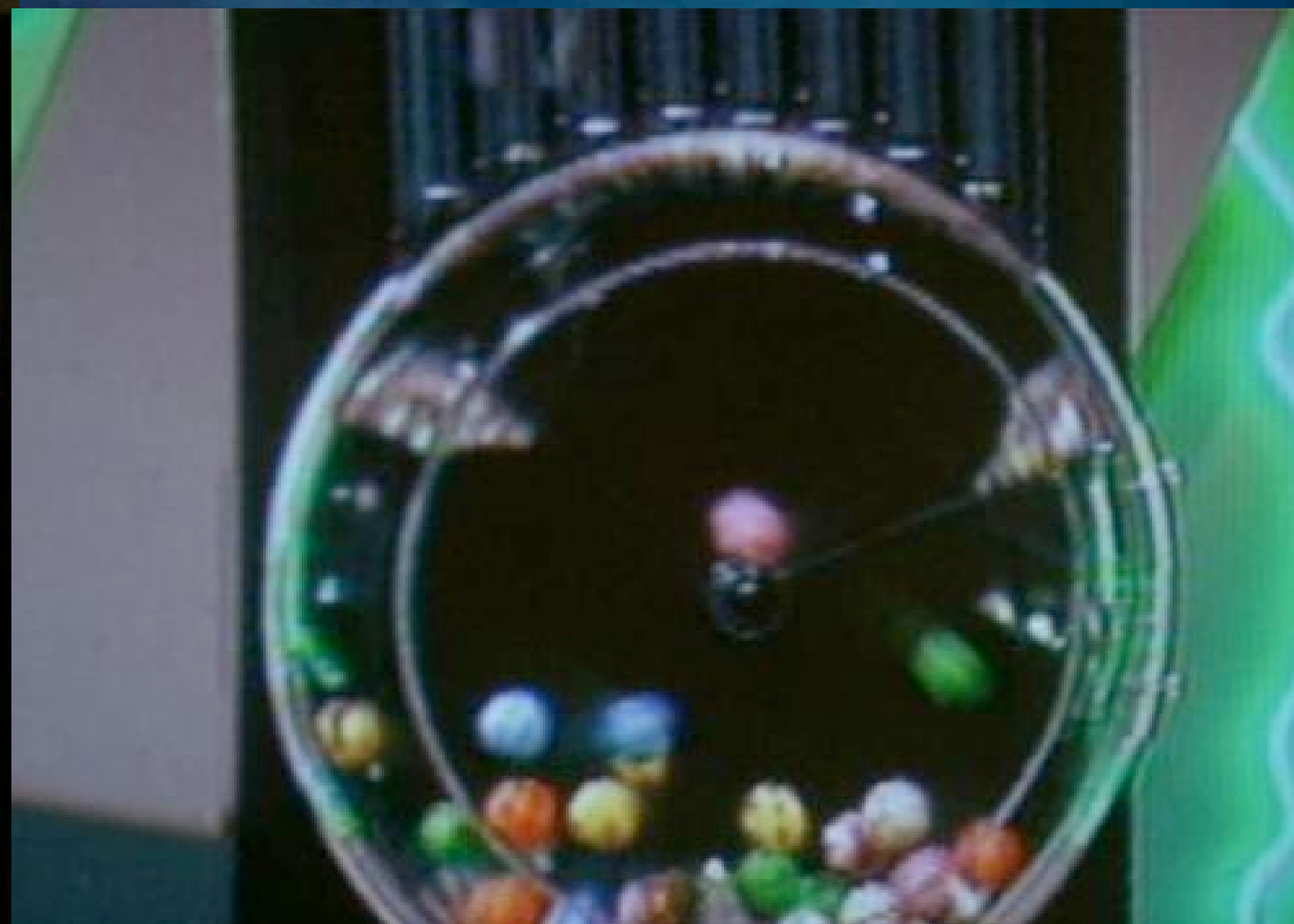
Global warming starts to disrupt world climate

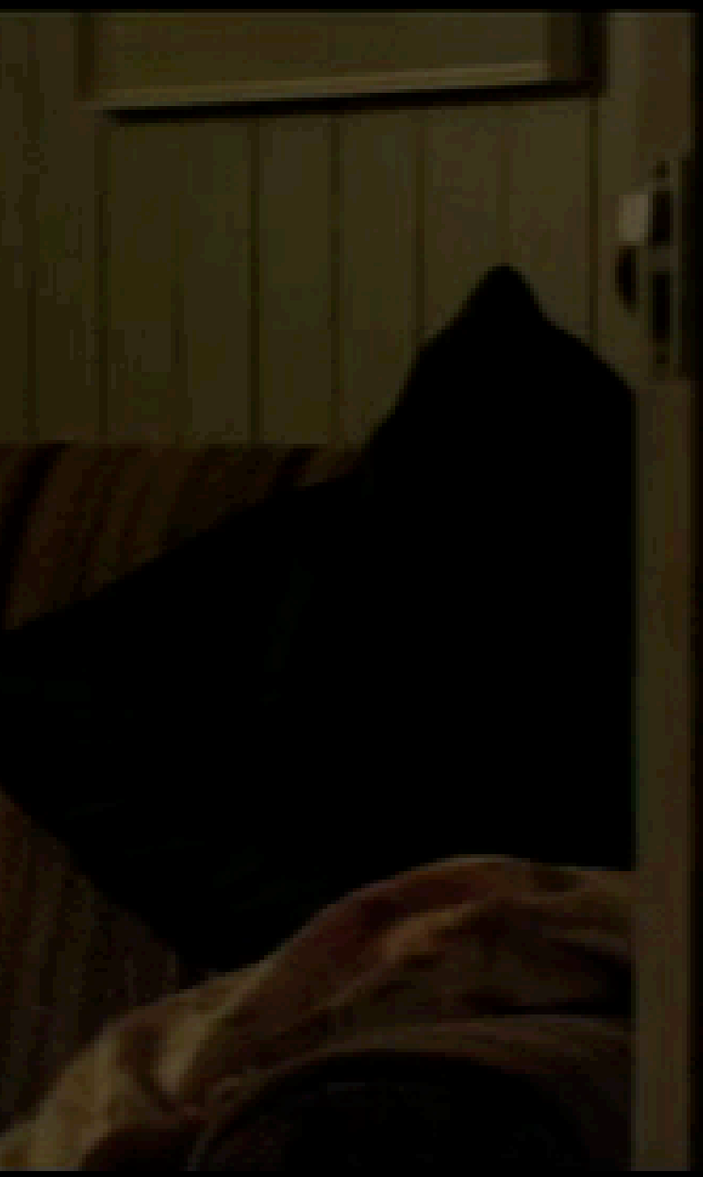
Bangkok Post, Thailand - 1 hour ago

Exeter, England _ Evidence is growing that **global warming** is already starting to disrupt the world's delicately-balanced climate system, and the damage will **...**

Predictability IX







From : Waking Ned Devine

Predictability X

Experiment: iterate the map $T(x) = 4\,x\,(1-x)$
as well as the map $S(x) = 4\,x - 4\,x^2$

Untitled-1

```
In[29]:= T[x_] := 4 x (1 - x);
         Last[NestList[T, 0.3, 60]]

Out[30]=
         0.207807
```

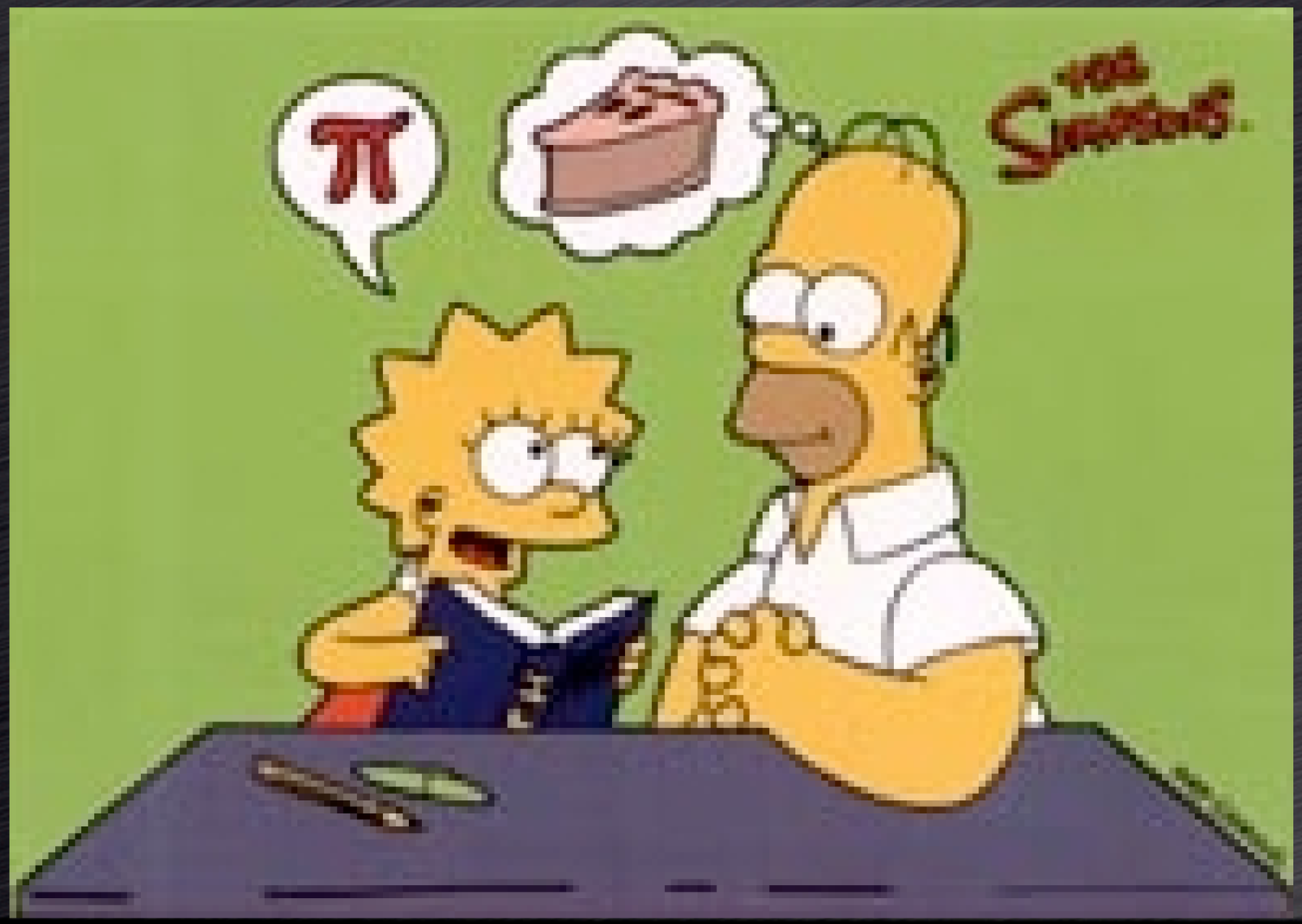
```
In[27]:= S[x_] := 4 x - 4 x^2;
         Last[NestList[S, 0.3, 60]]

Out[28]=
         0.31637
```

200%

A flapping of butterfly wings in Rio de Janiero

can produce a tornado
in Texas several weeks later.

The mathematicians notion of a dynamical system



Back to the definition

A semigroup is a set G with an operation *
which is associative (u*v)*w=u*(v*w)

A semigroup G acts on a set X,
if for each t in G, there is a
transformation T(t) on X such
that T(t*s)(x) = T(t) T(s)(x)

A dynamical system (G,X)
is a semigroup G acting
on a set X.

While this is much too abstract and general for us and just about the contrary to the approach we use (focus on examples) it is useful to keep this in mind.

Two cases which matter for us:

G=0,1,2,3,4....

Maps like $T(x)=2x\,(1-x)$

define $T^n(x)$

G=[0, infinity)

Differential equations

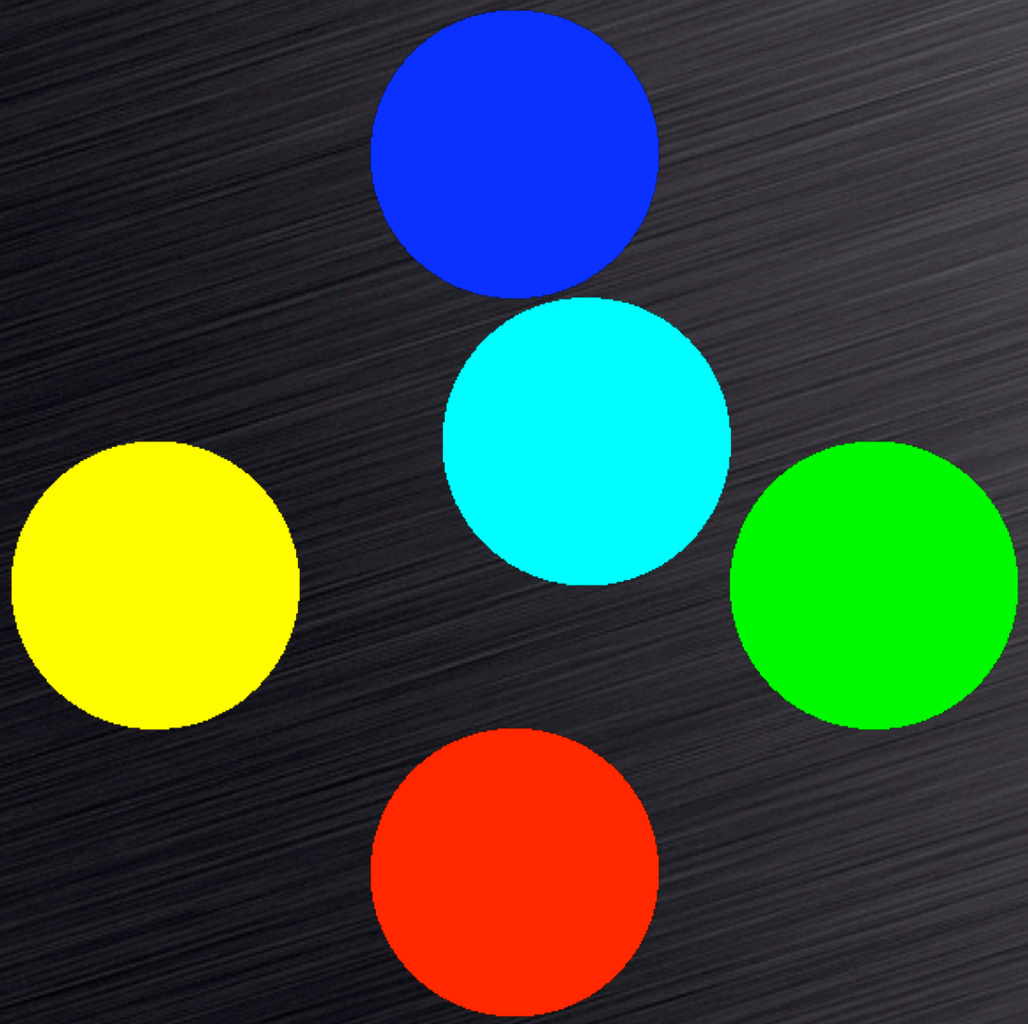
like $d/dt\ x = x\,(1-x)$

define flows $T^t(x)$

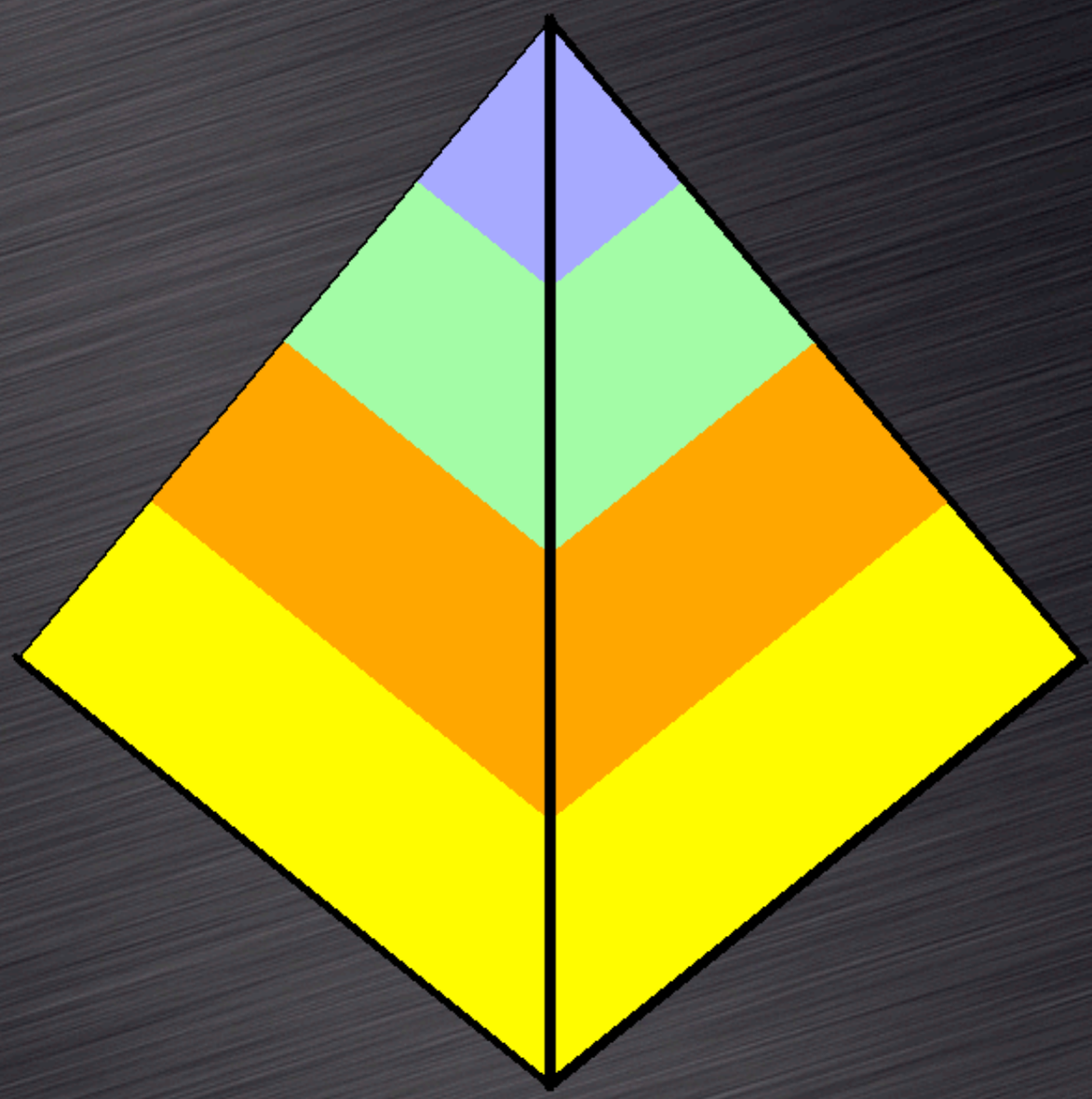
C: Organization of the course

- Case method
- Frequent, straightforward quizzes
- Focus on examples
- Final quiz and project

Case Method



Traditional Method



Quizzes

Instead of:

Homework

First Midterm

Second Midterm

Final

Q1

Q2

Q3

Q4

Q5

Q6

Q7

Q8

Final Quiz

Homework

Project of choice

Focus on Examples

- A good example is worth a thousand theories.
- Many examples are universal. What you see in the Henon map for example, you see in many dynamical systems
- The universality principle shows that measureable quantities can be independent of the system.

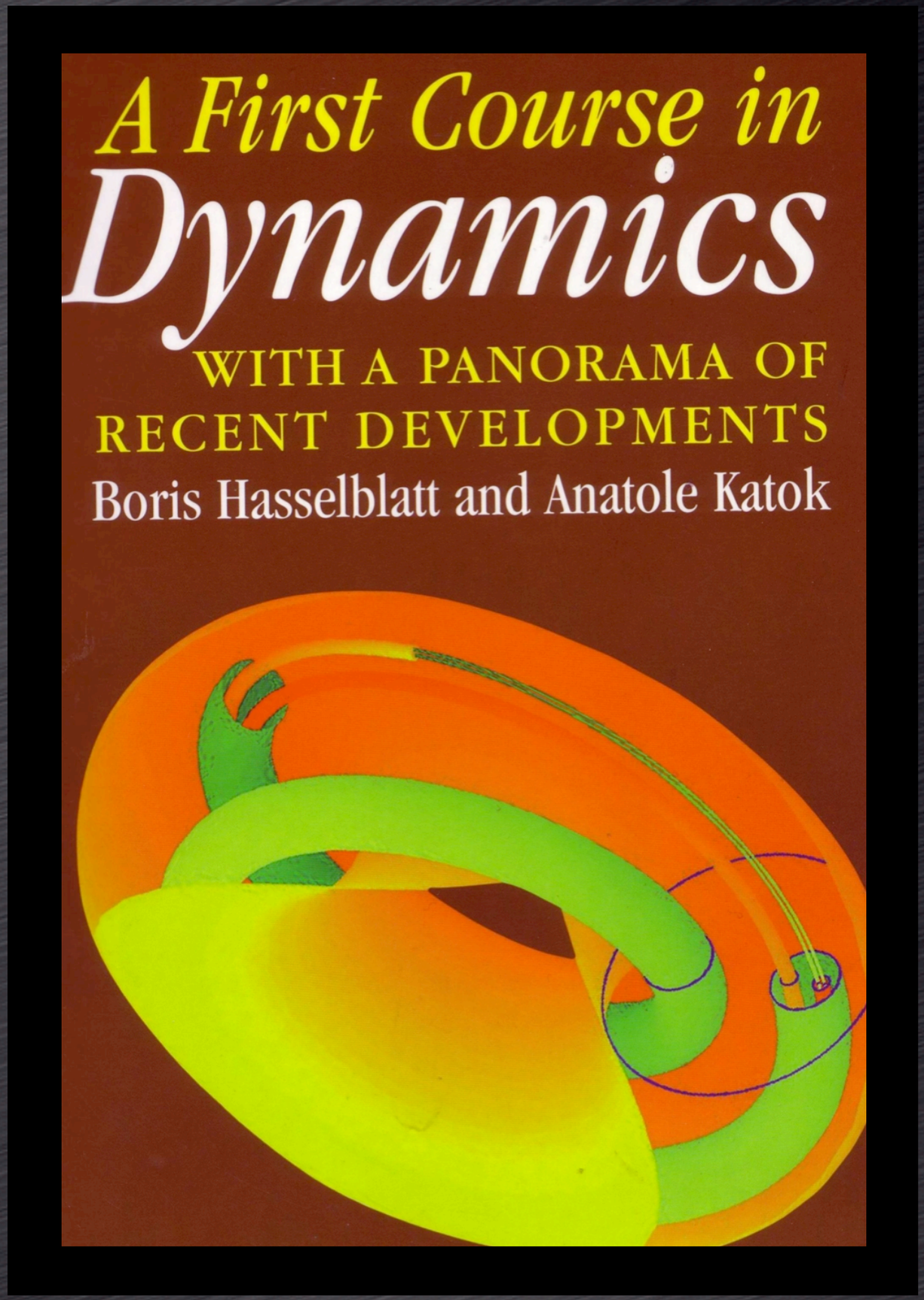
Final Project

- Suggesitions are online
- You can also chose something on your own

Reading text

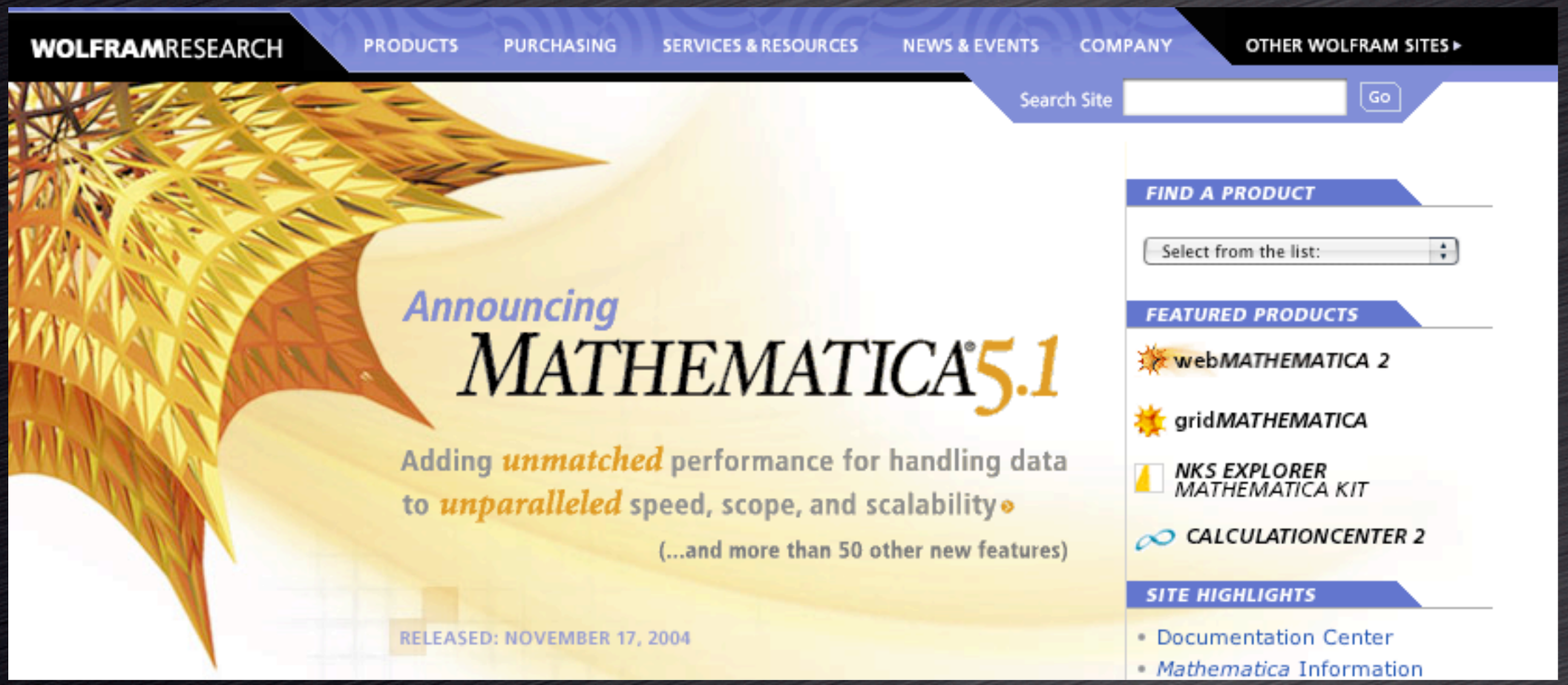
Paperback edition available at Coop

Besides the fact that it is an excellent book written by leading experts, the prize was also a reason to chose this book.



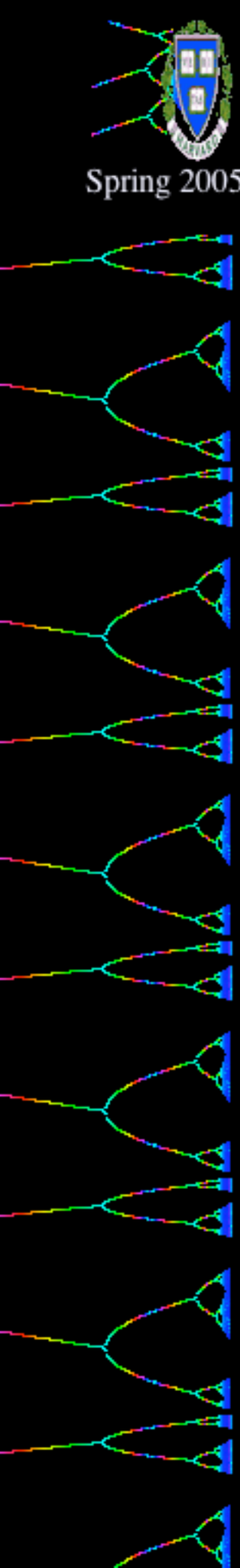
Software:

- If you want to redo some of the experiments done in class, download Mathematica, a computer algebra system. Harvard has a cite license.



D: Syllabus

We have 13 weeks. We will cover about 10 topics.



Spring 2005

Mathematics 118r Dynamical Systems Spring 2005

Course Head: Oliver knill
Office: SciCtr 434
Email: knill@math.harvard.edu

Syllabus

calendar:

```
-------------------------------------------------------
Su Mo Tu We Th Fr Sa   week no and events
-------------------------------------------------------
    +-----+-----+
    |     |     |
30 31  1  2  3  4  5    1  2. february start of lectures
 6  7  8  9 10 11 12    2
13 14 15 16 17 18 19    3
20 21 22 23 24 25 26    4  21. february presidents day
27 28  1  2  3  4  5    5  march
 6  7  8  9 10 11 12    6
13 14 15 16 17 18 19    7
20 21 22 23 24 25 26    8
27 28 29 30 31  1  2       april        spring break
 3  4  5  6  7  8  9    9
10 11 12 13 14 15 16    10
17 18 19 20 21 22 23    11
24 25 26 27 28 29 30    12
 1  2  3  4  5  6  7    13 may          end of classes
    |     |     |
    +-----+-----+
 8  9 10 11 12 13 14       reading period
15 16 17 18 19 20 21       19. start of exam period
22 23 24 25 26 27 28
-------------------------------------------------------
```

tentative syllabus:

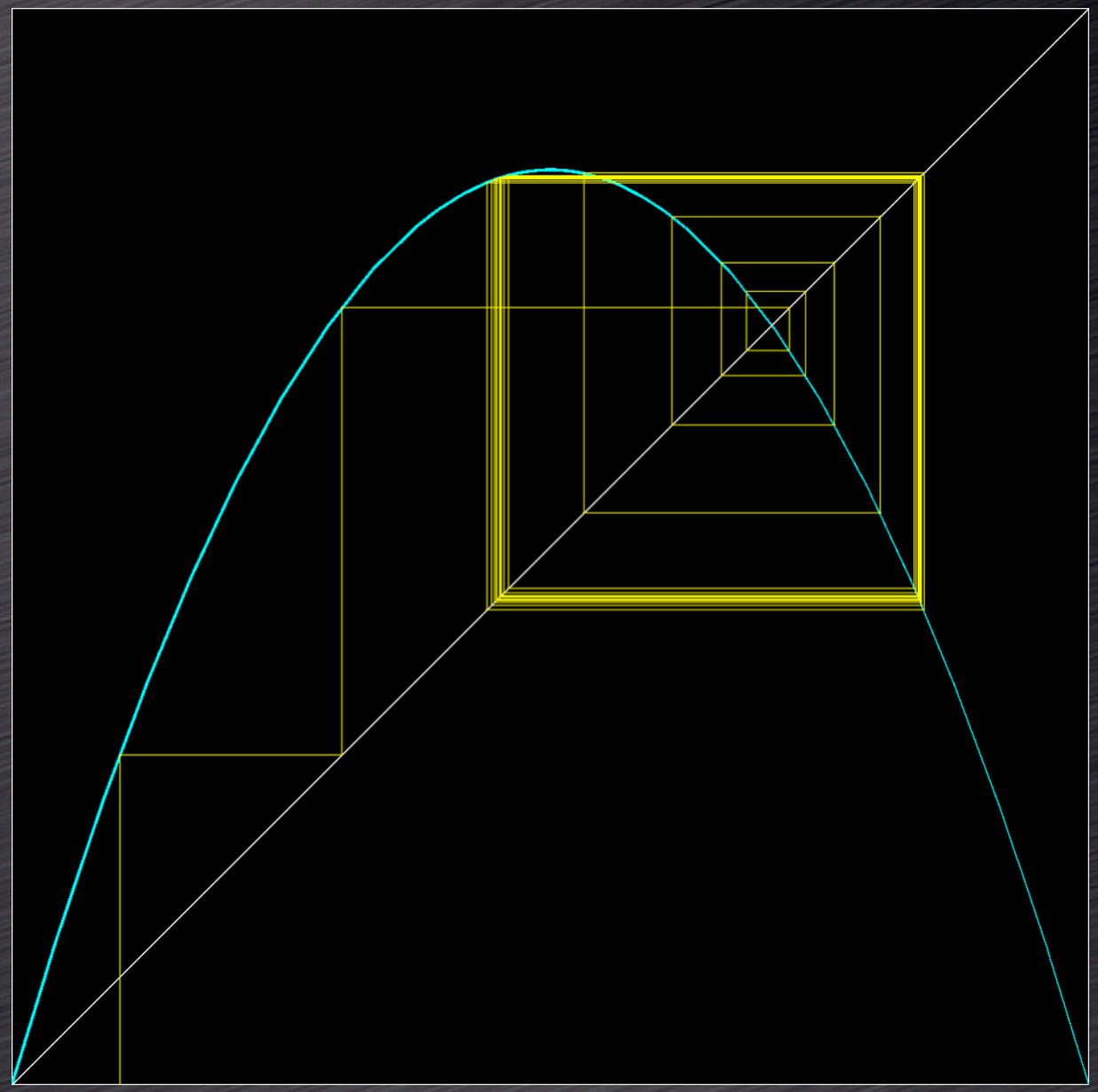
1. Week *Introduction:*
   Wed: Overview and organization of the course.
   Fri: Examples of dynamical systems
2. Week *Feigenbaum: maps in one dimensions*
   Mon: Maps on the interval
   Periodic points and their stability.
   Wed: Bifurcation of periodic points

Week 1 (this week):

- What are dynamical systems?
- Organization of the course.
- Syllabus
- Examples of dynamical systems

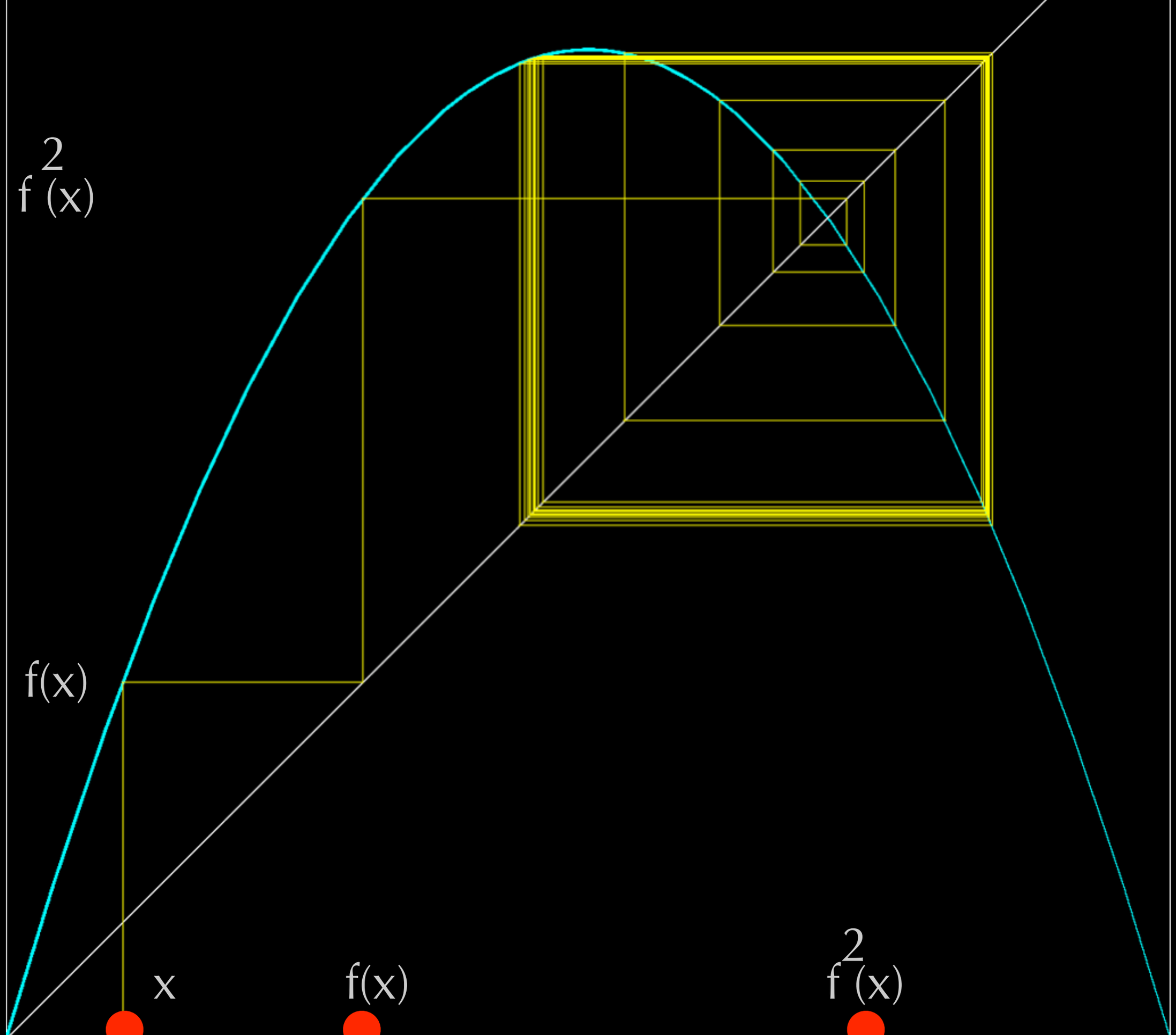
Week 2: maps in 1D

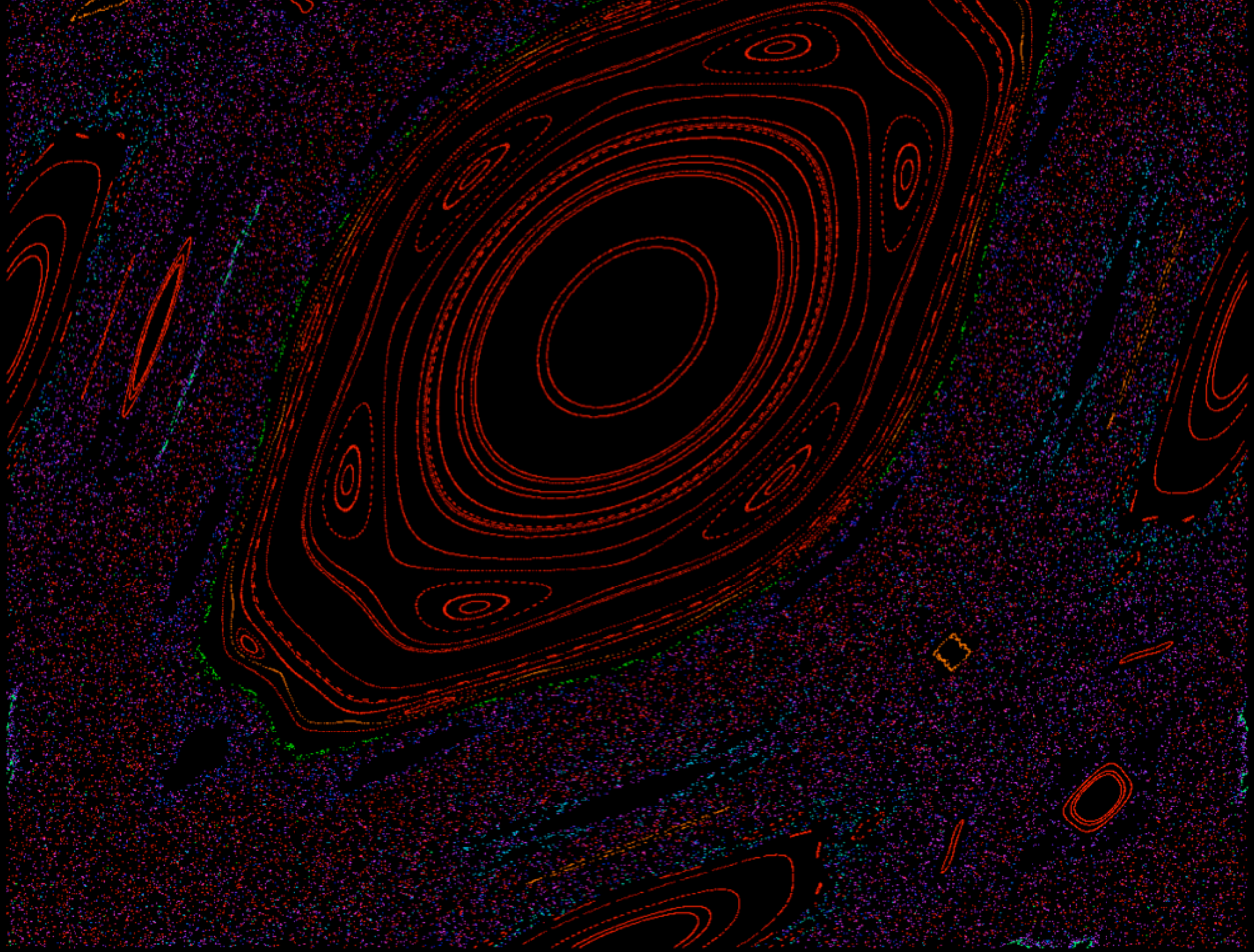
- The logistic map
- Periodic points and bifurcations
- Lyapunov exponents and chaos
- Dynamical zeta function

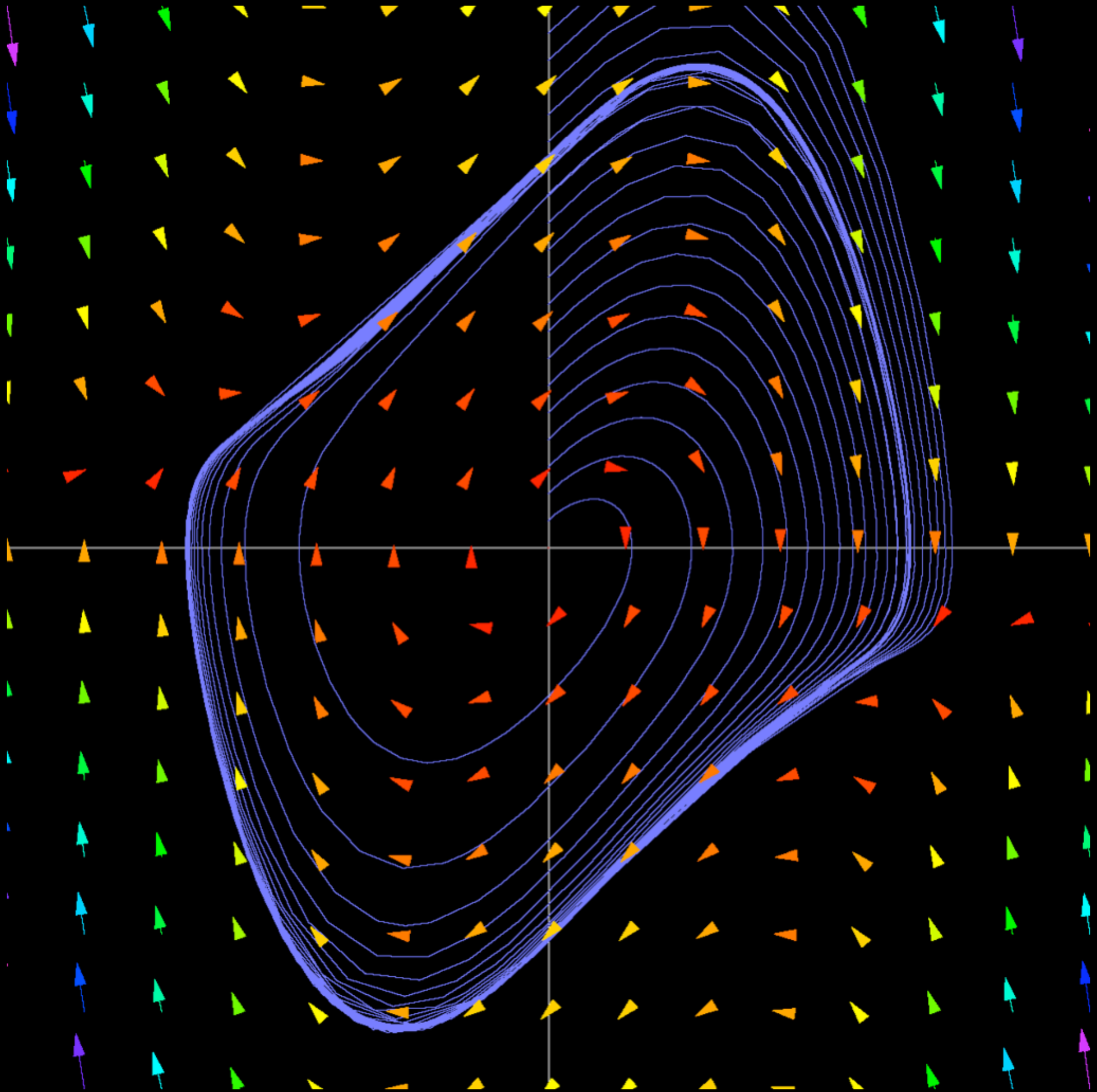


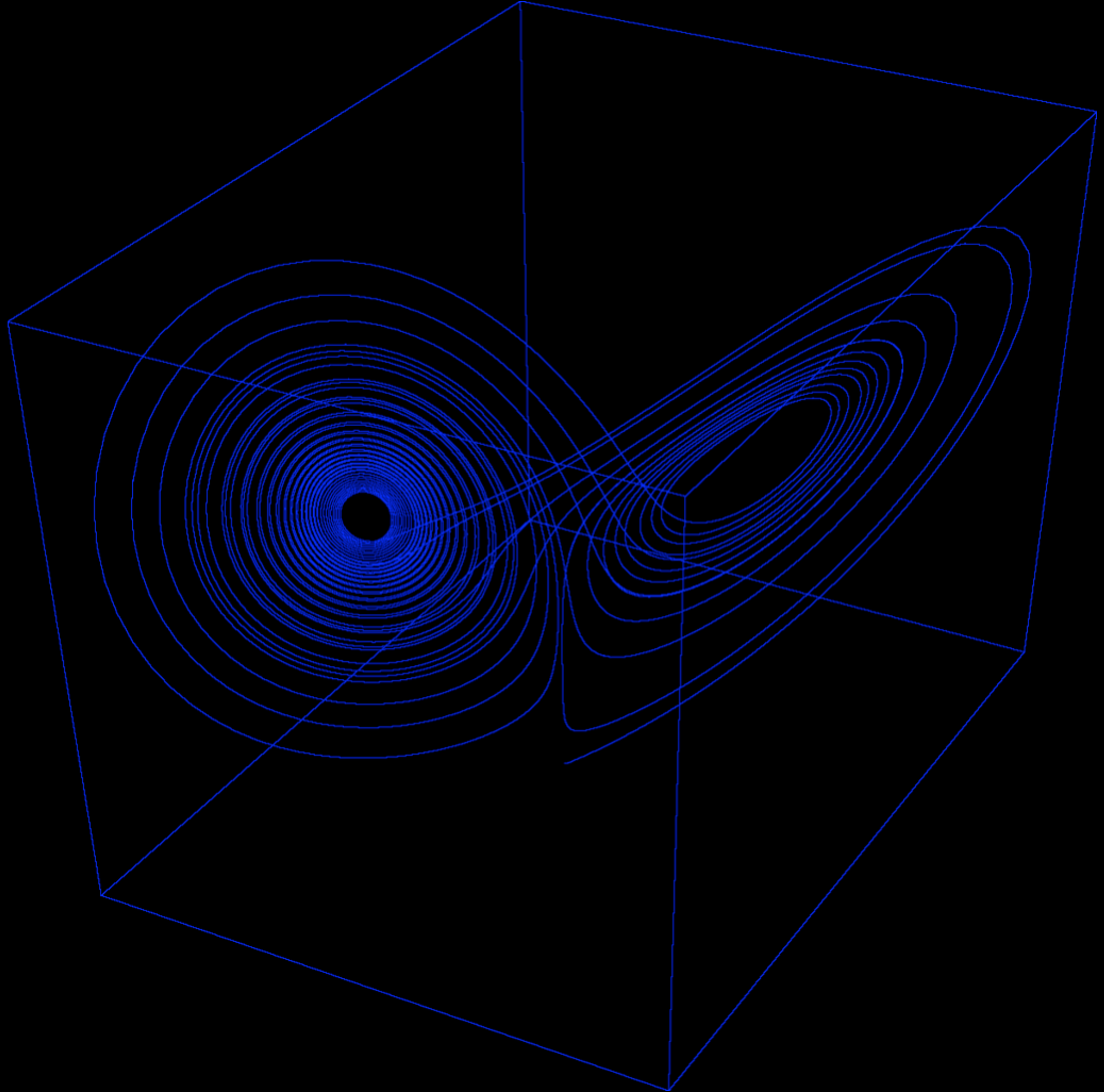
Lets just continue with pictures:

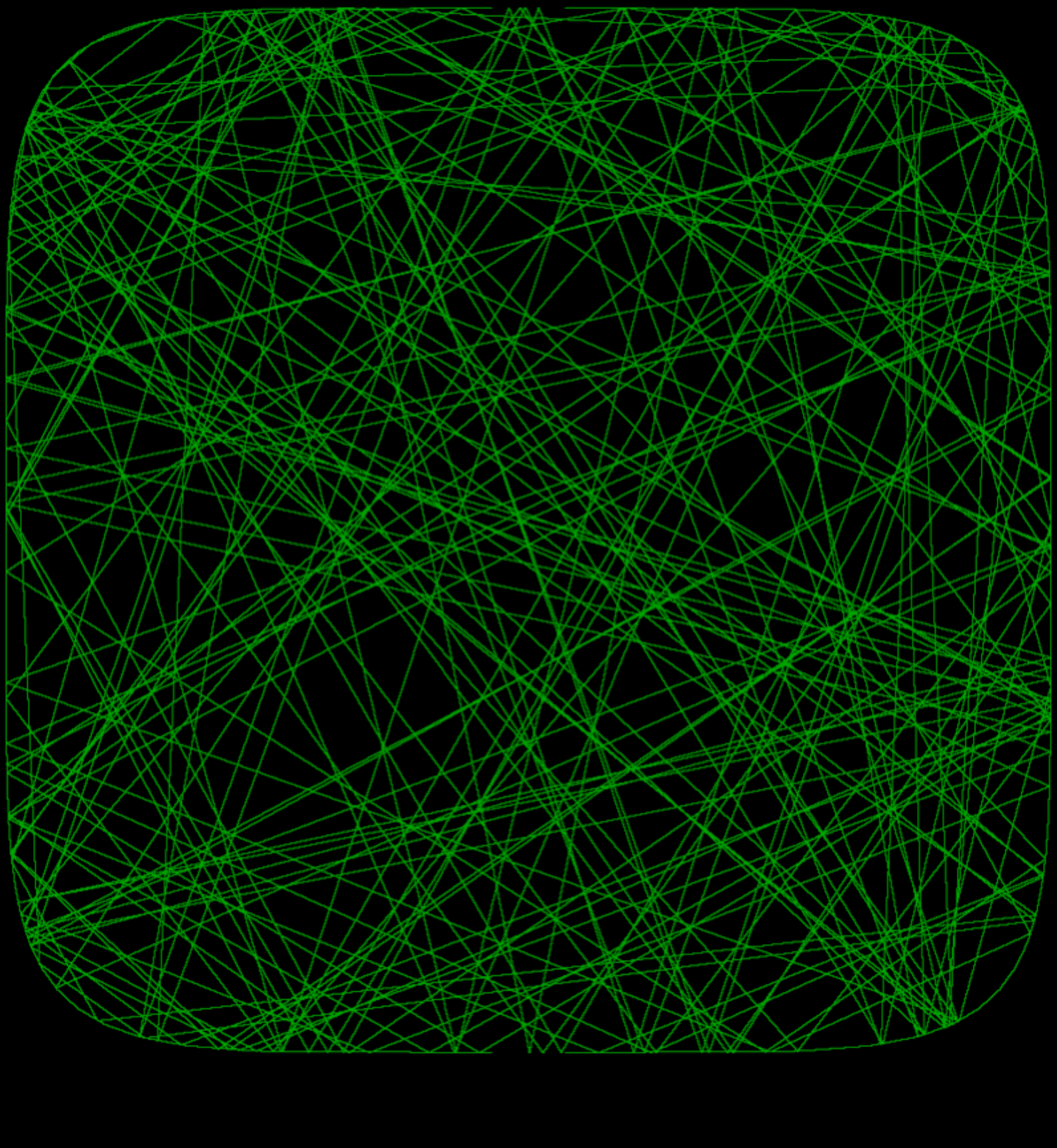
Next Friday,we will make an exhibition which is a bit more detailed. Right now, these are just pretty pictures.

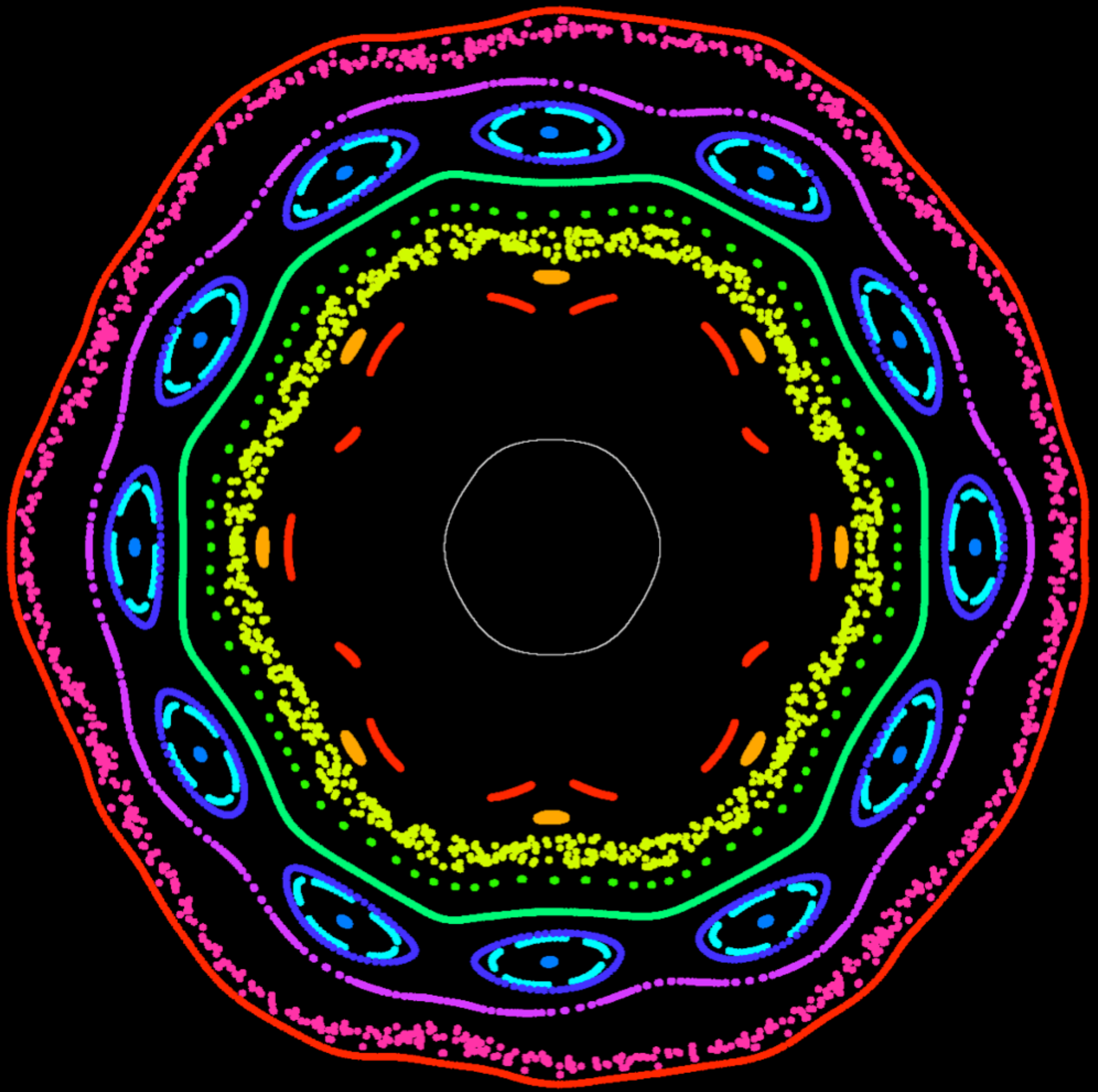

2
f (x)
f(x)
x
f(x)
2
f (x)

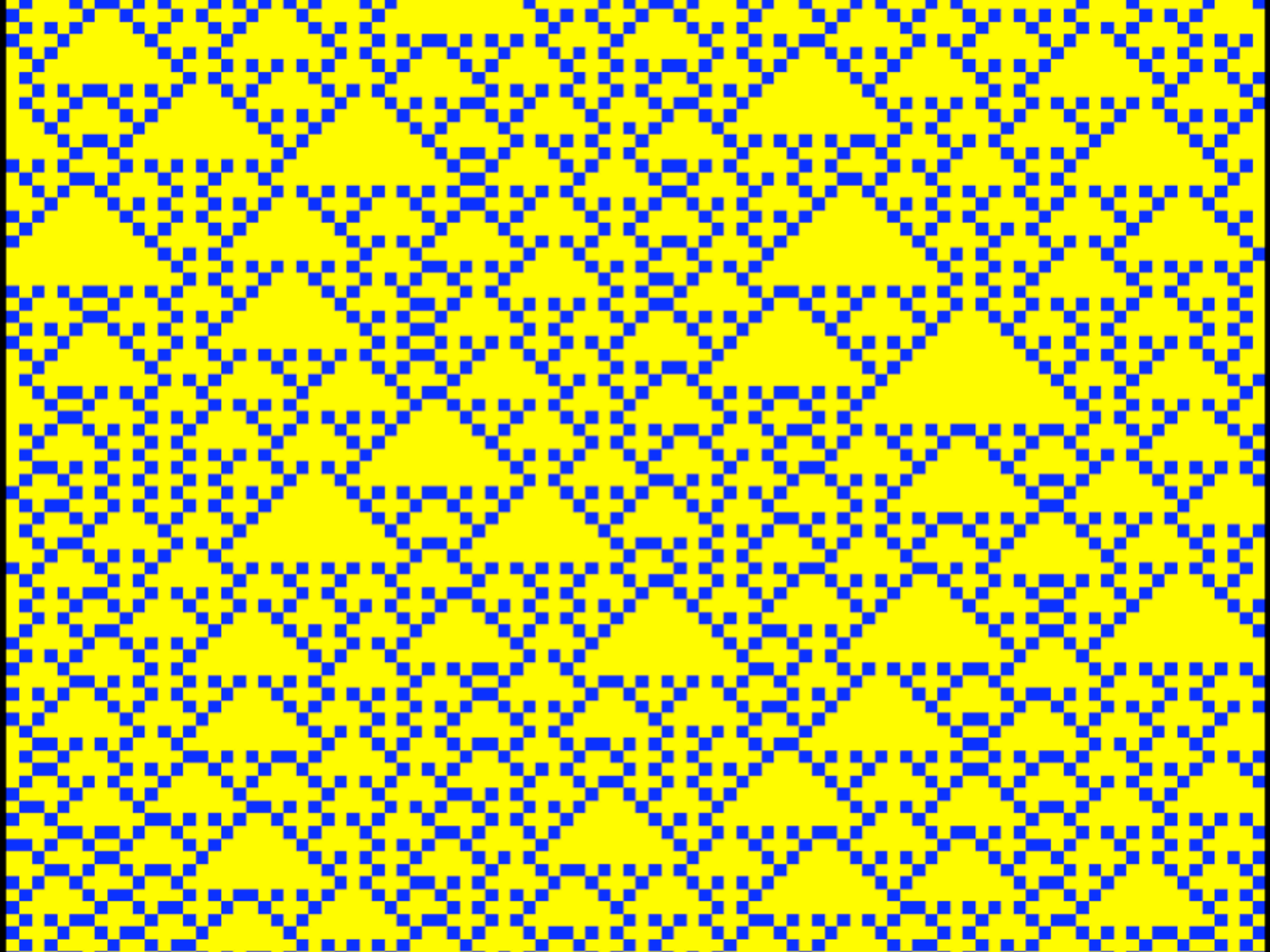


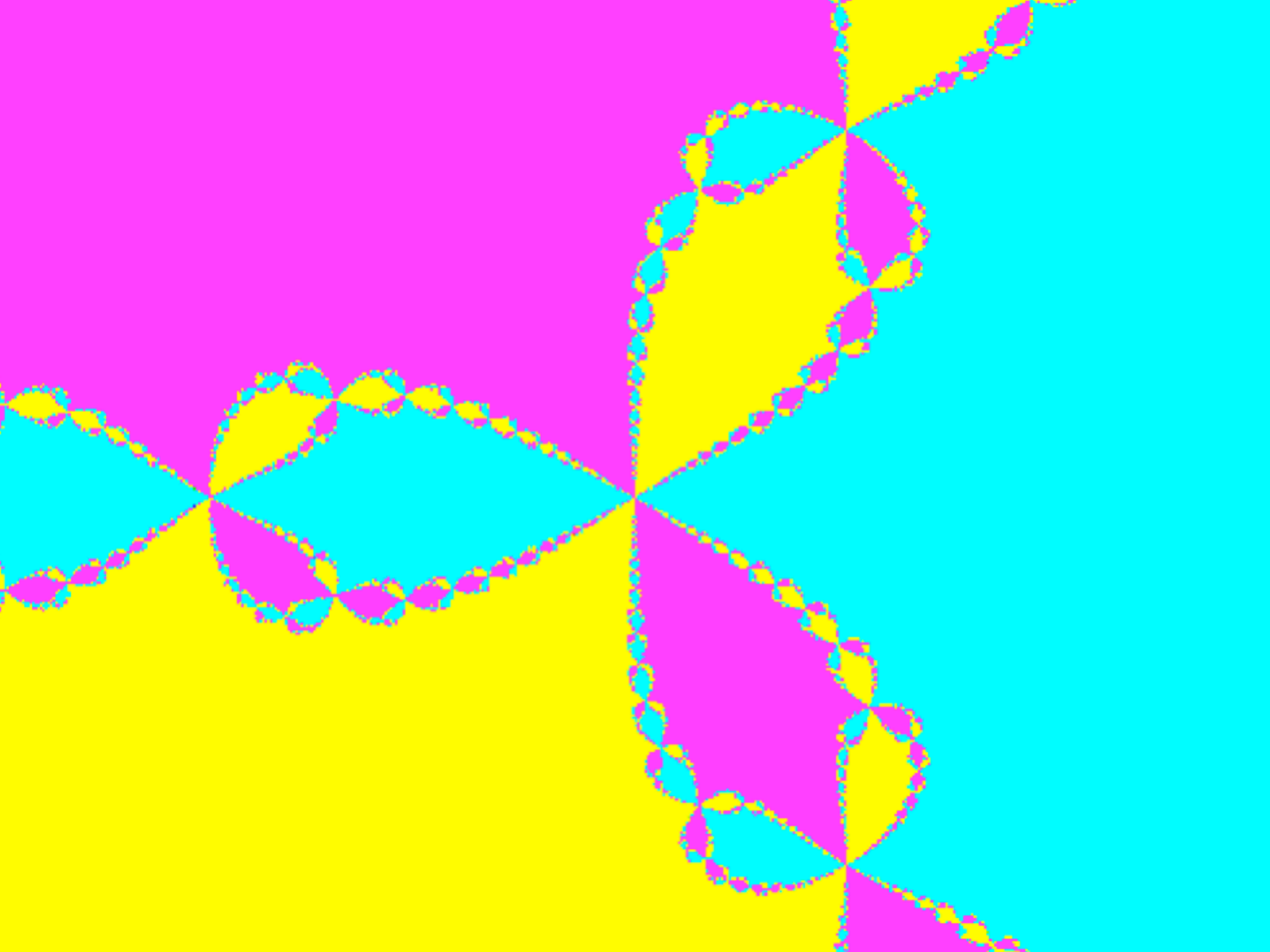




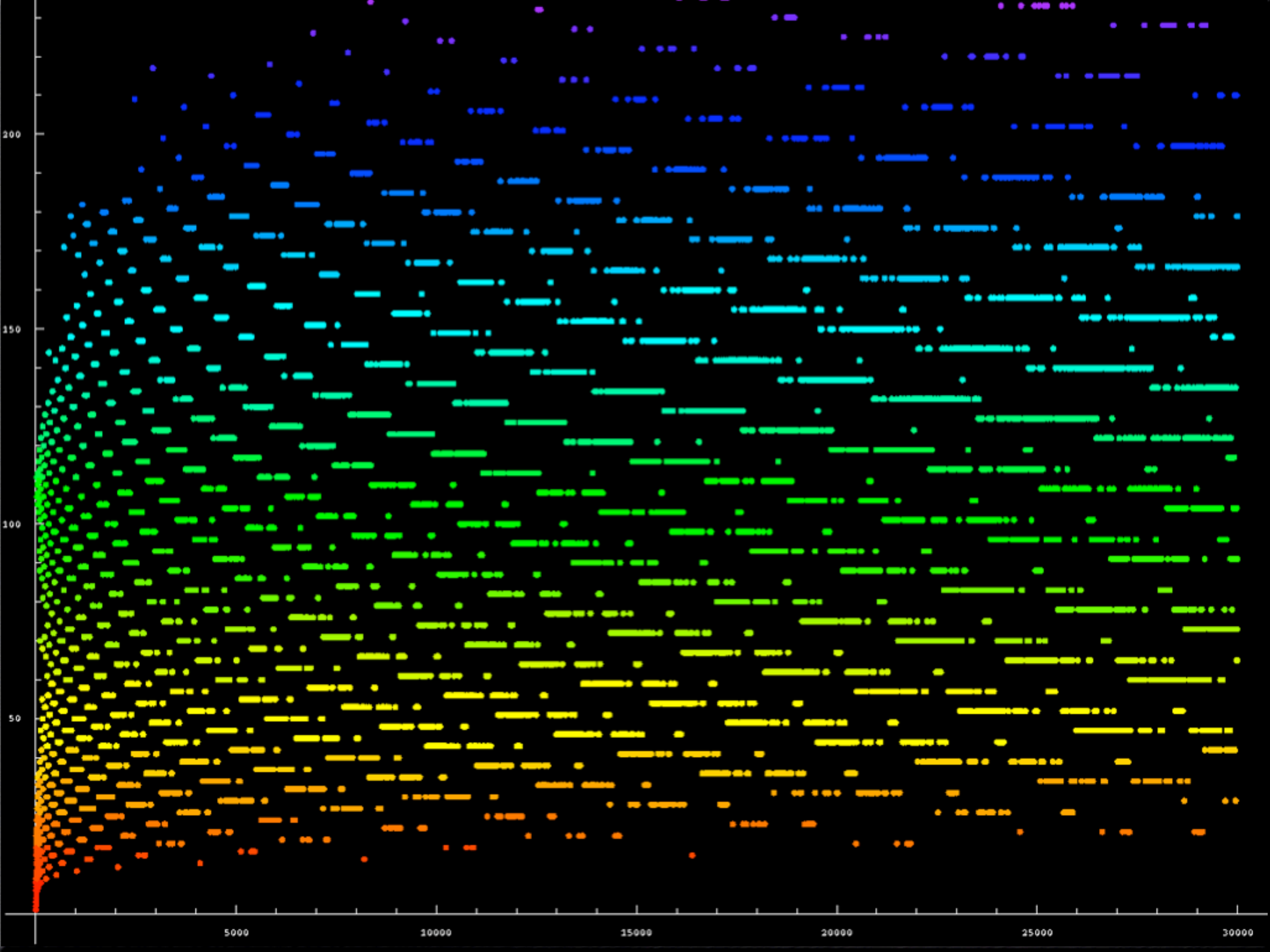









2
1.5
1
0.5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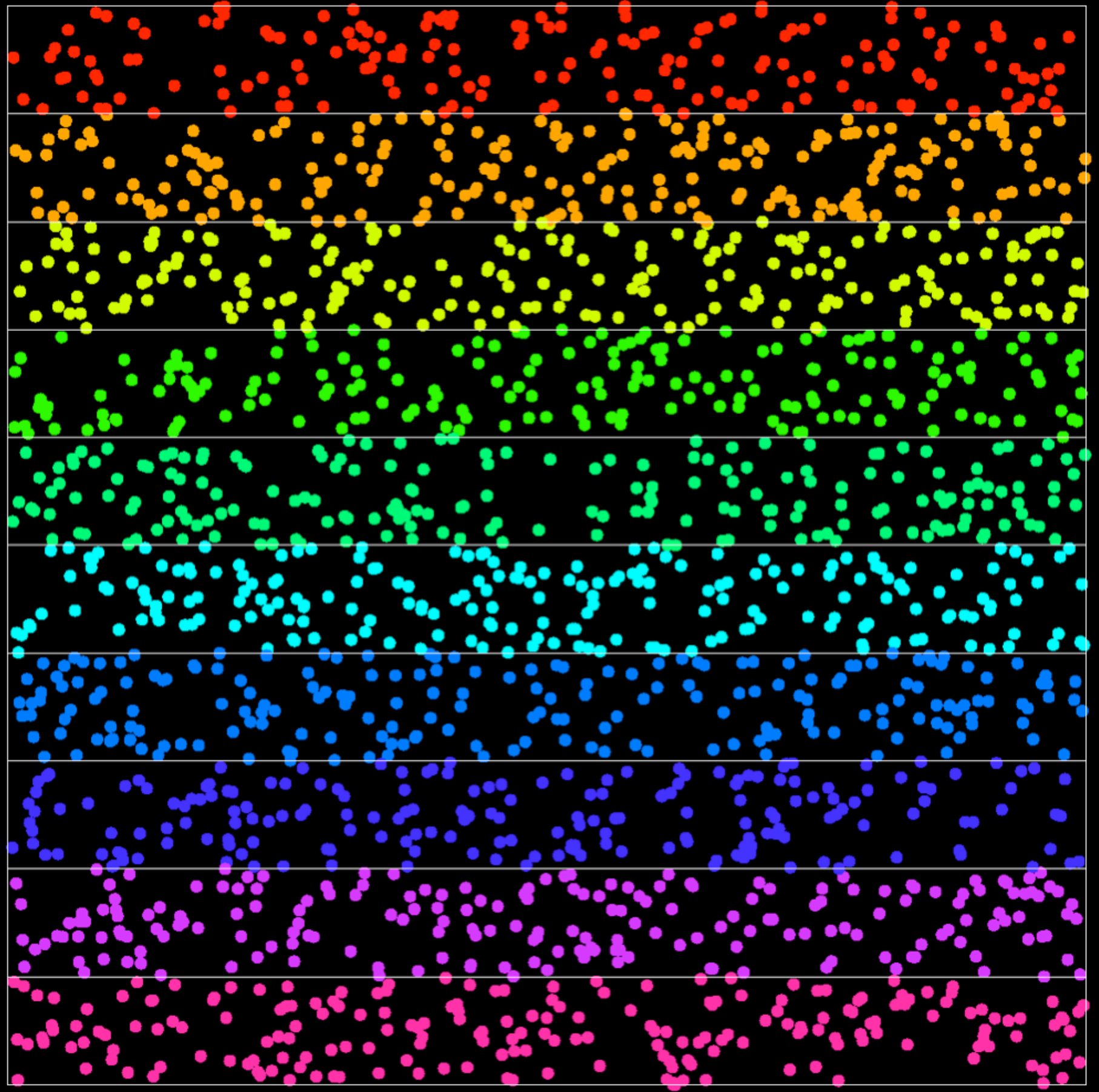

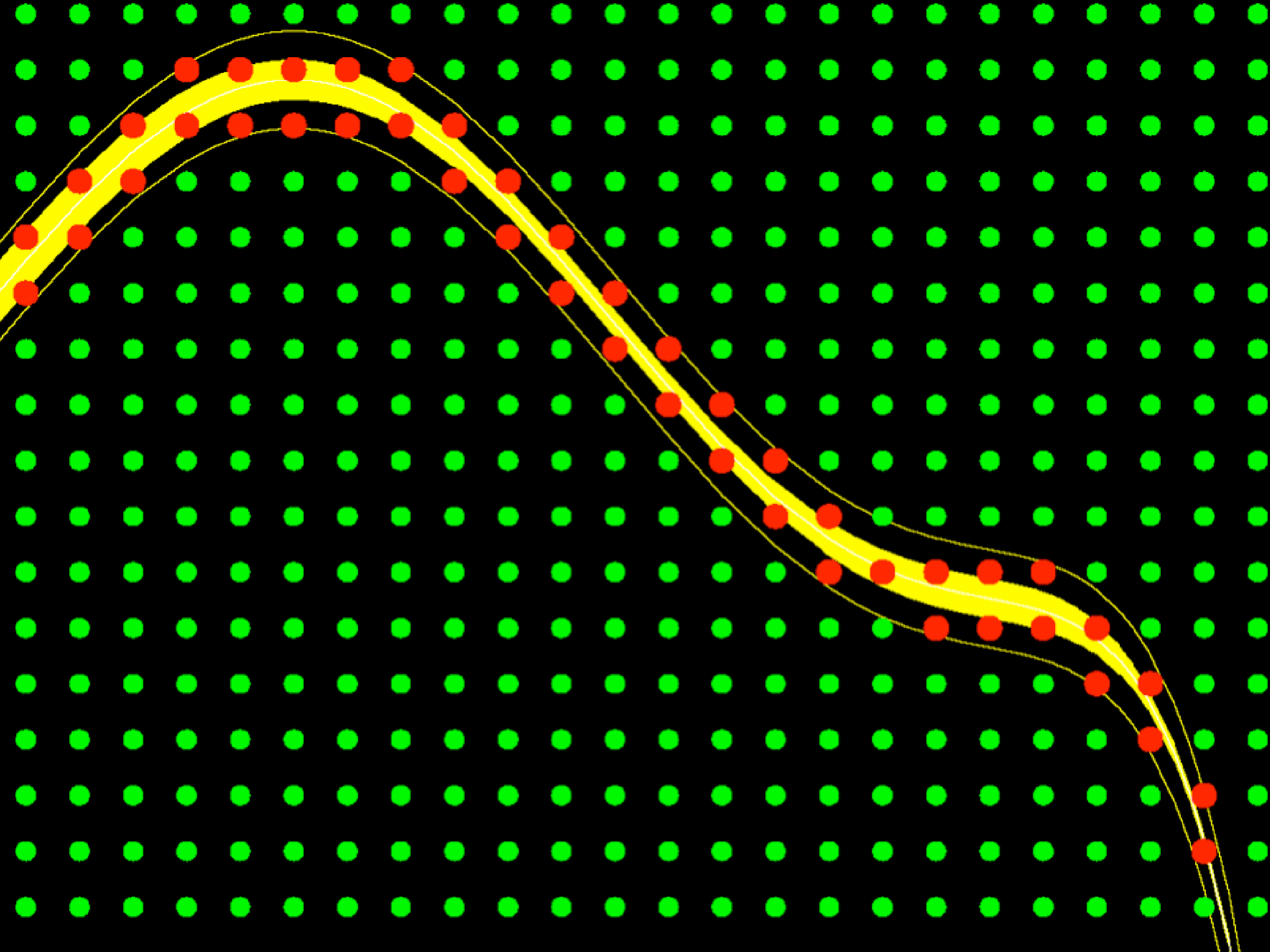

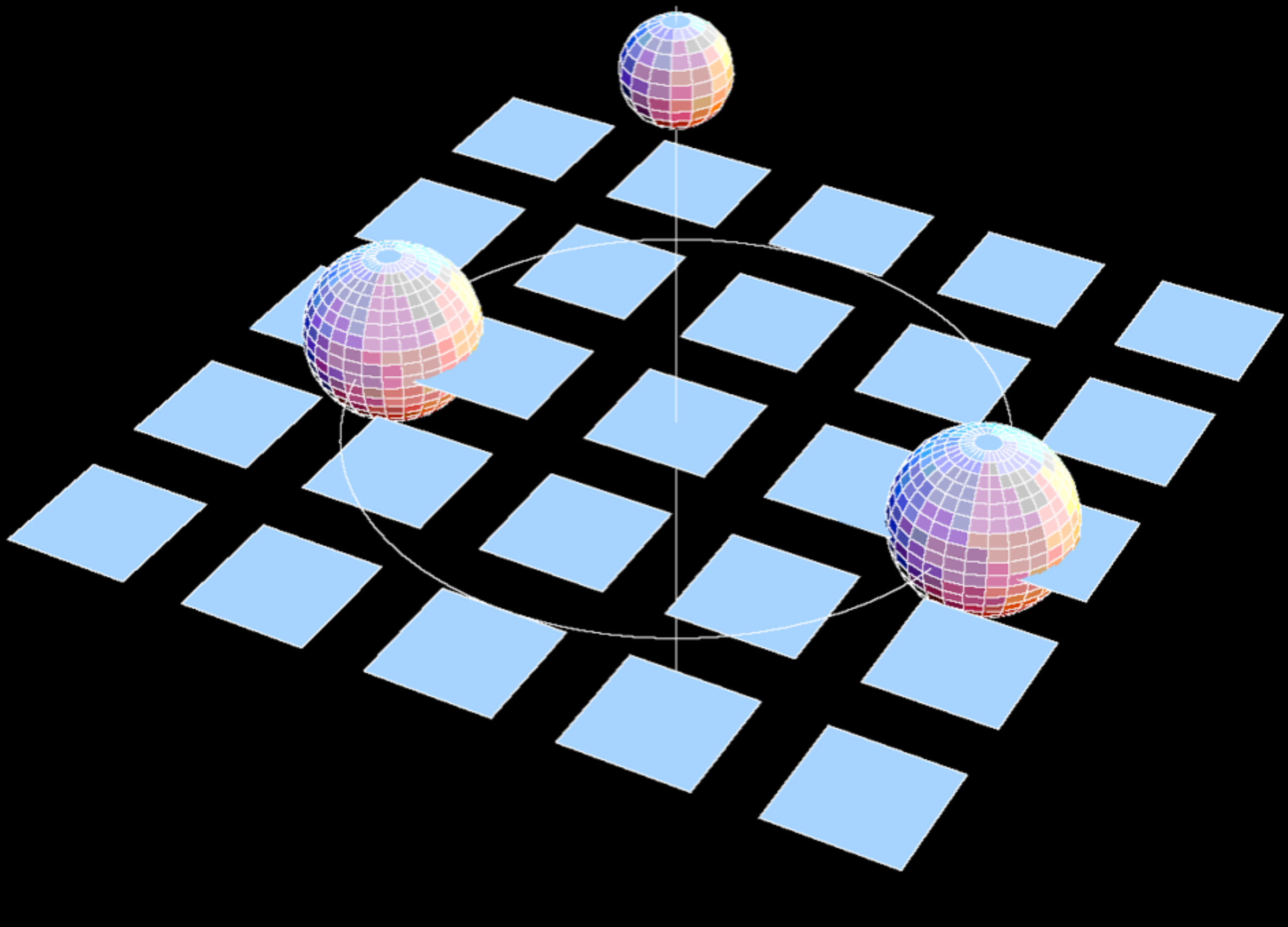

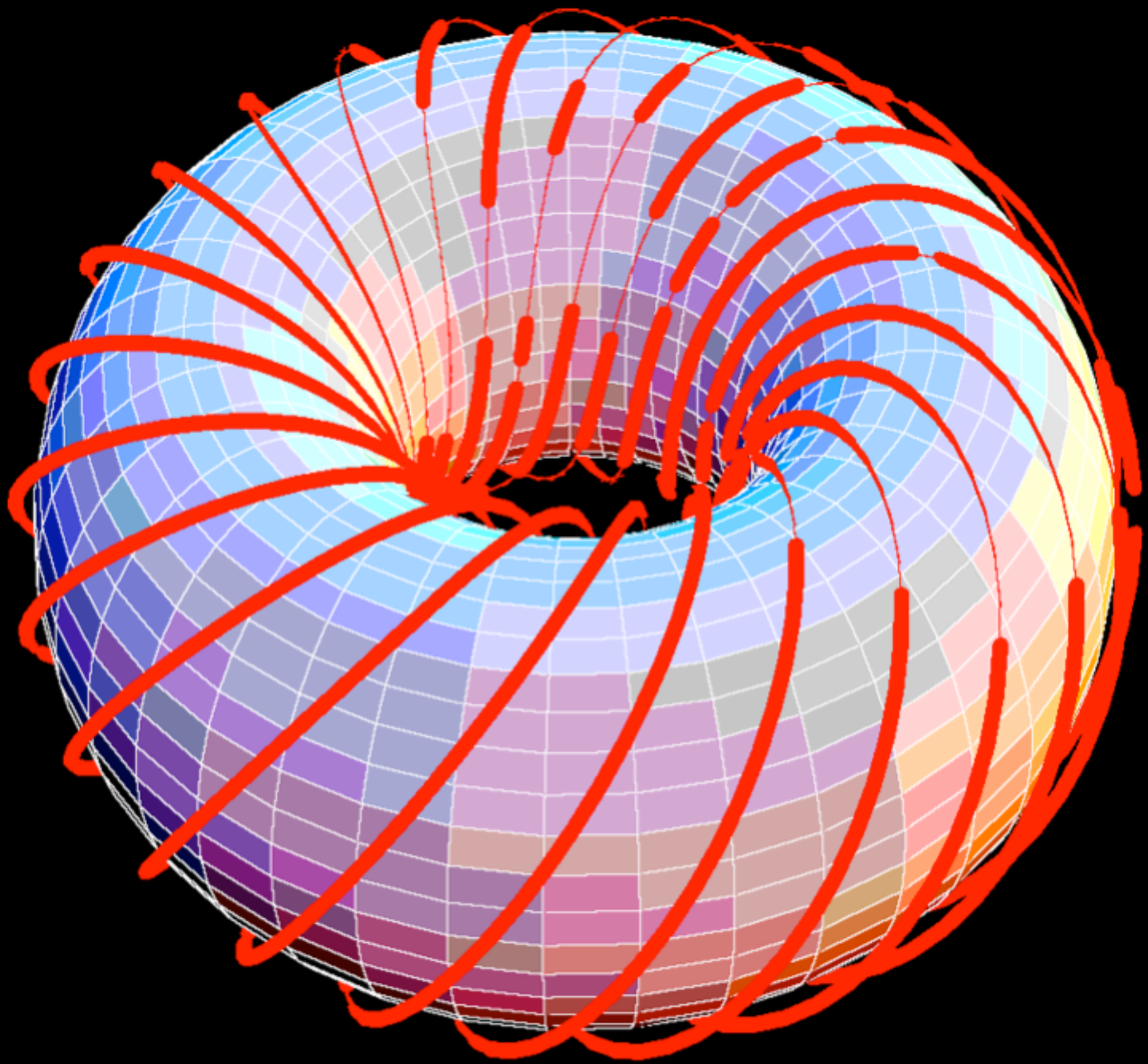

Thats all for today.

If you have questions, talk to me right now after lecture, or then this afternoon at 3 PM in Sci Center 434