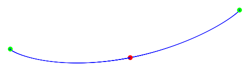
THE LORENZ SYSTEM II — **Math118, O. Knill**

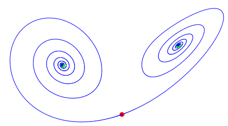
ABSTRACT. This is a continuation of the discussion about the Lorenz system and especially on the r dependence of the attractor.

OVERVIEW OVER BIFURCATIONS. We fix the parameter $\sigma = 10, b = 8/3$.

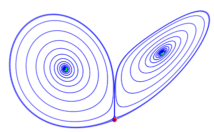
For $0 < r < 1$, the origin is the only equilibrium point and all points attracted to this point (you can find a proof in the book. At $r = 1$, a **pitchfork bifurcation** takes place. The origin becomes unstable and two stable equilibrium points appear. The picture shows the case $r = 1.5$.



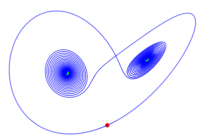
For $1 < r < 13.925$, the unstable manifold of the origin connects to the equilibrium points. The picture shows $r = 10$.



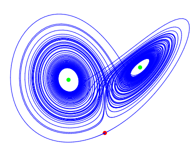
For $r = r_0 = 13.926$, the unstable manifold becomes double asymptotic to the origin.



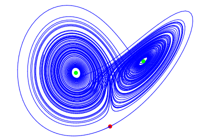
At the parameter r_0, two unstable cycles appear. For $13.926 < r < 24.06$, these cycles come closer to the fixed points $C^{\pm}$. The picture shows the parameter $r = 20$.



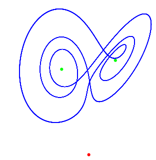
At the parameter $r = r_1 = 24.74 = 470/19$, the unstable cycles collide with the stable equilibrium points and render them unstable. This is called a **subcritical Hopf bifurcation**.



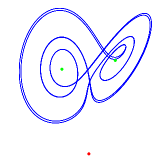
At the parameter $r = 28$, one observes the Lorenz attractor.



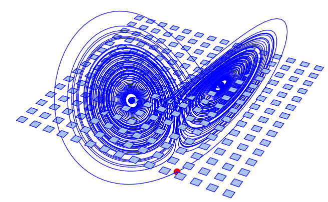
Between $r = 0.99524$ and $r = 100.795$, one observes an infinite series of period doubling bifurcations of stable periodic points (one has to start with the larger value and decrease r). These bifurcations are analogue to the Feigenbaum scenario. The picture shows the parameter $r = 100$.



Here we see the previous stable periodic cycle doubled. The parameter is $r = 99.7$. The period doubling scenario leads to the same Feigenbaum constant as one can see in the one dimensional logistic map family.



RETURN MAP. A good Poincare map is part of the subplane $z = r - 1$. This plane contains the equilibrium point $C^{\pm}$. These points are fixed points of the return map.



HISTORICAL. Lorenz carried out numerical investigations following work of Saltzman (1962). The Lorenz equations can be found in virtually all books on dynamics. We consulted:

- C. Sparrow, "The Lorenz equations: Bifurcations, chaos and strange attractors, Springer Verlag, 1982
- Strogatz, "Nonlinear dynamics and Chaos", Addison Wesley, 1994
- Dynamical systems X, Encyclopadia of Mathematics vol 66, Springer 1988
- Dennis Gulick, Encounters With chaos, Mc Graw-Hill, 1992
- Clark Robinson, Dynamical systems, Stability, Symbolic Dynamics and Chaos, CRC priss, 1995