MIDTERM

1. Discuss the dynamics of the family $f_\alpha(x) = x^3 - \alpha x$ for $-\infty < \alpha \leq 1$.

2. Assume that $f : X \to X$ and $g : Y \to Y$ are topologically conjugate via $h : X \to Y$. Show that

a. If p is a periodic point of prime period n of f, then $h(p)$ is a periodic point of prime period n of g. If p is eventually periodic for f, then $h(p)$ is eventually periodic for g.

b. If p is an attracting periodic point of period n for f with stable set $W(p)$, then $h(p)$ is attracting for g with stable set $h(W(p)) = \{y \,|\, y = h(x), \text{for some } x \in W(p)\}$. In particular, if there is a open neighborhood U of p such that for every $x \in U$, $f^{in}(x) \to p$ monotonically as $i \to \infty$, then there is a open neighborhood V of $h(p)$, such that for every $y \in V$, $g^{in}(y) \to h(p)$ monotonically as $i \to \infty$.

c. If p is a repelling periodic point of f, then $h(p)$ is a repelling periodic point of g.

d. If $f : X \to X$ has a dense orbit in X, then $g : Y \to Y$ has a dense orbit in Y.

e. Assume further that X and Y are bounded closed intervals. show that if f has sensitive dependence on initial data on X, then g has sensitive dependence on initial data on Y.

3. Show that $f(x) = x^3 + x/2$ is C^1 structurally stable.

4. Prob. 4 in section 1.6.

Due 10/31